Game theory and risk-based leveed river system planning with noncooperation

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Abstract
Optimal risk-based levee designs are usually developed for economic efficiency. However, in river systems with multiple levees, the planning and maintenance of different levees are controlled by different agencies or groups. For example, along many rivers, levees on opposite riverbanks constitute a simple leveed river system with each levee designed and controlled separately. Collaborative planning of the two levees can be economically optimal for the whole system. Independent and self-interested landholders on opposite riversides often are willing to separately determine their individual optimal levee plans, resulting in a less efficient leveed river system from an overall society-wide perspective. We apply game theory to simple leveed river system planning where landholders on each riverside independently determine their optimal risk-based levee plans. Outcomes from noncooperative games are analyzed and compared with the overall economically optimal outcome, which minimizes net flood cost system-wide. The system-wide economically optimal solution generally transfers residual flood risk to the lower-valued side of the river, but is often impractical without compensating for flood risk transfer to improve outcomes for all individuals involved. Such compensation can be determined and implemented with landholders’ agreements on collaboration to develop an economically optimal plan. By examining iterative multiple-shot noncooperative games with reversible and irreversible decisions, the costs of myopia for the future in making levee planning decisions show the significance of considering the externalities and evolution path of dynamic water resource problems to improve decision-making.

1. Introduction
Levees can increase channel flow capacity to protect adjacent areas from potential floods, but they can fail by overtopping and various intermediate structural failure modes [Tung and Mays, 1981a; Hui et al., 2015]. In flood-prone river basins, individual landholders sometimes lack incentives to cooperate in planning local levees with other landholders upstream, downstream, or across the river. Historically, noncooperation in such conflicting situations caused economically inefficient and damaging outcomes [e.g., Kelley, 1989; Barry, 1997]. Collaboration among individual landholders can produce more system-wide economically optimal levee systems, with less damage overall and to all landholders, but collaboration does not always occur.

Flood risk to economic activity is the likelihood of lost property and economic disruption due to flooding times the magnitude of losses, and is measured by economic metrics as direct and indirect costs [Traver et al., 2014]. Rare and unpredictable events could cause extreme consequences [Taleb, 2010]. Risk in flood management is normally the probability of failure multiplied by the consequences of failure, summed over all possible events [Hashimoto et al., 1982]. Levees can decrease but cannot eliminate the likelihood of flooding and flood risk, given levee failures under various conditions. Risk-based optimization for levees and leveed river systems planning has long been examined to formally include various uncertainties [Van Dantzig, 1956; Tung and Mays, 1981a]. Optimal planning of a leveed river system minimizes the overall expected annual costs on both riverbanks and reaches, which includes annualized construction cost and expected (residual) annual damage cost [Hui et al., 2015].

Different river system levee plans would have different flood risk distributions. A symmetric system of levees has two identical levees (and failure probabilities) on opposite riversides, while an asymmetric system of levees has a lower levee that is more likely to fail. Croghan [2013] discussed the economic flood risk transformation and transference among landholders, finding that total flood risk could be reduced from transferring...
risk from a high-cost urban side to a lower-valued rural side of a river. Hui [2014] theoretically and numerically proves that an asymmetric leveed river system can increase the overall economic optimality by transferring flood risk across the river and reducing system-wide costs. However, system-wide optimal solutions are not necessarily acceptable for all stakeholders [Read et al., 2014]. In asymmetric leveed river systems, the required flood risk transfer for system-wide cost optimization can increase individual costs. Therefore, compensation might be needed for the flood risk transfer to make the solution socially stable or acceptable. Various transaction costs and political barriers often prevent such compensation [Kelley, 1989; Croghan, 2013].

Game theory that examines how independent and self-interested individuals interact with each other can help in analyzing the strategy of each landholder for leveed river system planning involving two individuals on opposing riversides [Hui et al., 2015]. Game theory has been applied in many water conflict problems [Carraro et al., 2005; Zara et al., 2006; Parrachino et al., 2006]. Madani [2010] has reviewed game theory applications to water resource management. It emphasizes differences between outcomes predicted by game theory and solutions proposed by optimization methods that inherently assume perfect cooperation among all parties [Madani and Hooshyar, 2014]. Water engineers often easily understand cooperative game theory, since the solutions are sometimes similar to system-wide optimization that tends to address the conflicting goals of a system [Madani, 2010]. However, noncooperation in water conflicts is common and tends to be more stable, with poorer literature and knowledge of application [Madani, 2010]. Individual parties may not accept a socially optimal solution in practice and leave the negotiation when dissatisfied with the solution, due to the mismatch between social-optimality and stability [Read et al., 2014]. This paper applies noncooperative game theory to river system levee planning using risk-based optimization to analyze potential ways to guarantee a system-wide economically optimal solution and the costs of decision-making myopia (from myopic or short-sighted view for the future) [Pinches, 1982; Madani and Hipel, 2012; Madani and Dinar, 2012b]. Game theory could derive conditions needed to reach a system-wide economically efficient leveed river system, when decision makers on both riversides are rational. How to compensate for flood risk transfer to incentivize cooperation can be determined by comparing results for risk-based river system levee planning with different types of conditions.

The rest of this paper is organized as follows. Section 2 describes risk-based optimization and a simple game theory framework for a river levee system. Section 3 briefly reviews the economically optimal river system levee planning to minimize overall expected system-wide cost cooperatively. Following sections discuss the application of different noncooperative game conditions, with no collaboration of parties involved. Section 4 applies one-shot noncooperative game theory to the river system levee planning problem. Section 5 then analyzes river system levee planning as an iterative multiple-shot noncooperative game where decisions are reversible, while section 6 analyzes such a game where decisions are irreversible. Section 7 discusses the limitations of the proposed game theoretic analysis and section 8 concludes with key findings.

2. Analysis Framework

Heights of each levee on opposite riversides are the decisions in this risk-based river system levee planning problem. Here only overflow levee failure is considered, assuming no intermediate geotechnical levee failures [Eigenraam et al., 2014; Zhao et al., 2014; Hui, 2014]. Other levee planning related parameters are set by standards and are identical for two river banks. Land uses behind each river bank determine the potential economic damage.

This study examines an idealized simple leveed river channel, with one levee on each opposite riverside [Tung and Mays, 1981b]. In Figure 1, given a general trapezoid cross section, each levee is characterized with a crown width Bc, a landside slope α, a waterside slope β, and a height H (H1 and H2 represent heights on each riverside respectively). The leveed river system could have symmetric (or identical) opposing riverside conditions, or have asymmetric (or different) opposing riverside conditions. For example, both riversides can be rural areas with the same potential damage, or rural Riverside1 with smaller potential damage and urban Riverside2 with greater potential damage.

2.1. Risk-Based Optimization for the Whole Leveed River System

Risk-based optimization for levee planning normally minimizes the expected annual total costs (TC), which includes expected (residual) annual damage cost (EAD) and annualized construction cost (ACC) [Van...
Flood risk is calculated in this way for each landholder and the whole system, assuming the same risk aversion. Alternative approaches incorporating risk aversion are available to calculate the flood risk with probabilistic outcomes, for example, to minimize the maximum risk \cite{Wald, 1950}, where further study can examine the impact of risk aversion.

Considering only overflow levee failure, flood failures of a leveed river system solely depend on the heights of two levees. Varying relationships between the two levees are shown in Figure 1 to illustrate where flood damages possibly occur. The system has four simple potential failure outcomes during a major flood:

1. Levee1 and Levee2 fail simultaneously, symmetric levees \((H_1 = H_2)\) (Figure 1a);
2. Levee1 or Levee2 fails with a 50\% likelihood, symmetric levees \((H_1 = H_2)\) (Figure 1a);
3. Levee1 fails first if it is lower, asymmetric levees \((H_1 < H_2)\) (Figure 1b);
4. Levee2 fails first if it is lower, asymmetric levees \((H_1 > H_2)\) (Figure 1c).

The height difference between the asymmetric levees should exceed the least distinguished height (height increment \(\Delta H\)) that can completely transfer the flood risk to one riverside. This height increment would change under different conditions, and needs to be assigned case by case.

For the whole leveed river system, the objective to minimize \(TC(H_1, H_2)\) mathematically is:

\[
\text{Min } TC(H_1, H_2) = \text{ACC}(H_1, H_2) + \text{EAD}(H_1, H_2)
\]
\[ \text{ACC}(H_1, H_2) = (s \times V \times c + LC_1 + LC_2) \times \left[ \frac{r \times (1+r)^n}{(1+r)^n - 1} \right] \] (2)

\[ EAD(H_1, H_2) = \int_{Q_c(H_1, H_2)}^{\infty} D(Q) \times P_q(Q) \times dQ = D_c \times [1 - F_c(Q_c(H_1, H_2))] \] (3)

where \( r \) is the real (inflation-adjusted) discount rate, \( n \) is the planning lifetime of levees (years), \( s \) is a multiplier for administrative cost, \( c \) is the construction cost per unit volume, \( V = L \times \left[ Bc \times (H_1 + H_2) + \frac{1}{2} \left( \frac{2H_1 + 2H_2}{2} + \frac{1}{2} \right) \right] \) is the whole volume (trapezoidal) of two levees on the \( L \) long river, \( LC_1 = UC_1 \times A_1 \) is cost of purchasing land on Riverside1 to build the levee, with a cost per unit area of land \( UC_1 \), and the area of land occupied by Levee1 base \( A_1 = L \times \left[ Bc + \left( \frac{1}{2H_1} + \frac{1}{2H_2} \right) \right] \); \( LC_2 = UC_2 \times A_2 \), \( UC_2 \), and \( A_2 = L \times \left[ Bc + \left( \frac{1}{2H_1} + \frac{1}{2H_2} \right) \right] \) are corresponding parameters for Levee2. \( D(Q) \) is the damage costs function depending on peak flood flow \( Q \), assuming constant damage potential cost \( DP_1 \) of Riverside1 and \( DP_2 \) of Riverside2 for any levee failure. \( D_1 = DP_1 + DP_2 \), \( D_2 = \frac{1}{2}(DP_1 + DP_2) \), \( D_3 = DP_1 \), and \( D_4 = DP_2 \) are the damage costs of the four possible failure outcomes. With the above formulation, damage costs for the first failure always doubles the second failure given the same levee heights, so the inferior first failure outcome is not further compared. \( Q_c(H_1, H_2) \) is the leveed river channel capacity, determined by the lower one between \( H_1 \) and \( H_2 \). Manning’s equation is used here to convert flow with water stage. \( P_q(Q) \) and \( F_c(Q) \) are the probability density function (PDF) and cumulative distribution function (CDF) of annual peak flood flow \( Q \), here we assume the annual flood frequency is lognormal distributed.

Given the risk-based optimization model for overflow levee failure only, the optimal results (basically optimal levee heights \( H_1^*, H_2^* \) and optimal annual expected total costs \( TC^* = TC_1^* + TC_2^* \)) can be solved through enumeration or other search algorithms. In addition to satisfying the physical constraints, the optimal conditions include the First-order Necessary Condition that the first-order derivative of the objective is zero, and the Second-order Sufficient Condition that the second-order derivative should be nonnegative for minimization.

In single levee planning cases, where the other riverside never fails and flood risk cannot be transferred, an individually optimal height \( H^* \) for a single levee corresponds to an individually minimized \( TC^* \) [Hui et al., 2015]. Figure 2 shows the cases of single levee planning for two landholders with differing potential damages. As levee height increases, each landholder’s EAD is decreasing and its ACC is increasing. Therefore, the summed TC is first decreasing rapidly, dominated by the decreasing EAD, and then slowly increasing with larger ACC. For two landholders with different potential damages, the annualized construction costs are identical here for any given levee heights, while the expected annual damages differ proportionally. Due to smaller potential damage, Landholder1’s individually minimized \( TC^* \) and individually optimal \( H^* \) would be smaller than Landholder2 (\( TC_1^* < TC_2^* \) and \( H_1^* < H_2^* \)). \( H_1^* \) and \( H_2^* \) in Figure 2 are the upper bounds of Landholder1 and Landholder2’s possible best individual levee heights as discussed later.

### 2.2. A Simple Framework of Game Theory Application

Acting independently, each self-interested landholder would optimally determine the height of its own levee using risk-based optimization instead of considering the system-wide economic costs and impacts on others.

Game theory could analyze how the two conflicting landholders make their levee planning decisions, according to their own interests. Each landholder is considered as an independent player who makes its own height choice. Payoffs for possible outcomes of the game are TC for each individual landholder, including conditional EAD and ACC, where each landholder’s EAD is based on the relative two levee heights on opposite riversides [Hui et al., 2015].

The payoffs for Landholder1 are calculated with the function below.

\[ TC_1(H_1, H_2) = ACC_1(H_1) + EAD_1(H_1, H_2) \] (4)

\[ ACC_1(H_1) = (s \times V_1 \times c + LC_1) \times \left[ \frac{r \times (1+r)^n}{(1+r)^n - 1} \right] \] (5)
$EAD_1(H_1, H_2) = \begin{cases} 0, & \text{if } H_1 > H_2 \\ 0.5 \times \left[ \int_{Q_{c}(H_1)}^{\infty} DP_1 \times P_0(Q) \times dQ = 0.5 \times DP_1 \times [1 - F_0(Q_c(H_1))] \right], & \text{if } H_1 = H_2 \\ \int_{Q_{c}(H_1)}^{\infty} DP_1 \times P_0(Q) \times dQ = DP_1 \times [1 - F_0(Q_c(H_1))], & \text{if } H_1 < H_2 \end{cases}$

where $V_1 = L \times \left[ Bc \times H_1 + \frac{1}{2} \times \left( \tan \theta + \frac{1}{\tan \gamma} \right) \times H_2^2 \right]$ is the volume of Levee1 on the L long river reach; $Q_c(H_1)$ is the leveed river flow capacity, which depends on the lower $H_1$.

The payoffs for Landholder2 are similar, with corresponding parameters.

As one landholder’s EAD ($EAD_1(H_1, H_2)$ or $EAD_2(H_1, H_2)$) is a discontinuous function depending on relative levee heights, the payoffs ($TC_1(H_1, H_2)$ or $TC_2(H_1, H_2)$) are not continuous accordingly. This kind of payoffs drives the rational decision-making and determines the outcomes for various types of games that are analyzed below.

### 2.3. Illustrative Cases

A numerical example illustrates application of game theory to river levee system planning for various damage function and institutional conditions (USACE, 2006). For a symmetric river channel with identical riverside conditions, we assume both riversides are rural areas. For the asymmetric river channel system with different riverside conditions, we assume Riverside1 is rural area and Riverside2 is urban area. The parameters below affect results of the games, particularly the economic parameters, although general solution behavior and conclusions remain the same.

The example river is the Cosumnes River in California, with a mean annual peak flow of 100 m$^3$/s. Channel geometry and levee related parameters (Figure 1) are: channel width is 90 m including 60 m channel width and 30 m floodplain width, channel depth is 1 m, floodplain slope is 0.01, $\tan \alpha = 1/4$, $\tan \beta = 1/2$, $Bc = 10$ m, $c_{soil} = $ $30/\text{m}^3$, $r = 0.05$, $n = 100$ years, $s = 1.3$, longitudinal slope of the channel and floodplain is 0.0005, Manning’s roughness for the channel and floodplain is 0.05, and total levee length is $L = 3000$ m.

Where riverside conditions are identical, each rural riverside has a $1/\text{m}^2$ unit land cost and an assumed constant $8$ million damage cost if the protected area is flooded. If each levee is optimized individually and independently (disregarding conditions on the opposite riverside), the individually optimal height is $H_1^* = H_2^* = 3.1$ m corresponding to an individually minimized $TC_1 = TC_2 = $ $0.52$ million ($EAD_1 = EAD_2 = $ $0.16$ million, $ACC_1 = ACC_2 = $ $0.36$ million, and identical optimal levee failure probabilities $F_{Q_1} = F_{Q_2} = 0.020$).
Where riverside conditions differ, rural Riverside1 has the same economic parameters as above, with the same individually optimal levee planning and costs. Urban Riverside2 has a $3/m^2 unit land cost and an assumed constant $20 million damage cost if the urban area is flooded. Optimized as two single levees, urban Riverside2 would have $H_2^1=3.7$ m, $TC_2^1=9.69$ million, $EAD_2^1=0.22$ million, $ACC_2^1=0.47$ million, and $P_2^1=0.011$.


The system-wide leveed river system plan inherently assumes completely collaborative landholders for achieving an overall economically optimal solution [Madani and Hooshyar, 2014]. A collaborative river levee system is common when opposite riversides have the same landowners. But in case of separate ownership, cooperation involves practical complexities.

For identical conditions on opposite riversides, the least-cost plan for the whole system is symmetric when each levee fails with a 50% chance at the overflow height, sparing flooding at the opposite levee. Identical height values of the symmetric levees depend on height increment $\Delta H$, for example, $H_1^1=H_2^1=3$ m with $\Delta H=1$ m, and $H_1^1=H_2^1=2.7$ m with $\Delta H=0.1$ m. However, if both levees would likely fail under overtopping conditions (first potential failure outcome), asymmetric levees are economically optimal, despite identical riverside conditions [Hui, 2014; Croghman, 2013]. If flood damages differ, even slightly, on opposite riversides, the overall least-cost river system levee plan should be asymmetric, with the lower-valued riverside having a slightly lower levee. The lower-valued area then absorbs all residual flood risk by failing first (so long as failure is only by overtopping). The least-cost river levee system plan is where the height of urban Levee2 ($H_{ur2}^u=H_{ur1}^u+\Delta H$) being slightly taller than that of rural Levee1 ($H_{ur1}^r$), where the optimal height for rural Levee1 ($H_{ur1}^r=H_{ur1}^r$) also depends on $\Delta H$. Landholder2 benefits from (residual) flood risk transfer entirely to Landholder1 and from less construction cost than it would have with symmetric levees. And it may be necessary to compensate for the flood risk transfer to incentivize the system-wide economically optimal levee system plan.

Many types of economically efficient asymmetric river levee systems exist. Overflow weirs as well as other flood relief structures that allow flood flows to escape into a basin or bypass channel can be considered as lowered levees [Russo, 2010], transferring flood risk to the bypass channel. During a major flood, such a transfer also can occur by breaching the levee on the lower-valued riverside or raising the levee on the opposite riverside. Such a “levee battle” happened in the Mississippi floodplain as it went through significant reclamation of flood-prone tracts during the post Civil War boom, where the established cities like New Orleans battled the Mississippi. Due to many breaks in adjoining areas (Plaquemines Parish), rumors gradually arose that levees were purposefully weakened to save more valuable city property on the opposite riverside, since City officials worried about the safety of their own protective works [Barry, 1997]. A worse unexpected situation appeared after the 1849 flood on the Mississippi River that broke the levee at River Ridge, where uptown residents thought of strengthening the levee on their side, but those living on the opposite side threatened to prevent such measures by armed force. Similar levee battles have been fought elsewhere, including the Sacramento Valley in California [Kelley, 1989].

For the two landholders together, the overall benefit from collaboration, rather than competing, is $TC^c(H_1^c, H_2^c) - TC^c(H_1^c, H_2^c^*)$. $TC^c(H_1^c, H_2^c^*) - TC_1^c + TC_2^c$ is the overall equilibrium annual expected total cost under the competing noncooperative situation, which sums the individual equilibrium annual expected total cost to Landholder1 ($TC_1^c$) and Landholder2 ($TC_2^c$). $TC_1^c$ and $TC_2^c$ vary with different types of games and institutional arrangements, as discussed below. And $TC^c(H_1^c, H_2^c^*)$ is the system-wide minimum annual expected total cost for two landholders with cooperation.

4. One-Shot Noncooperative Planning

In noncooperative games, independent and self-interested players are separately making their own decisions and competing with each other [Madani and Hipel, 2012]. Typically, players would try to anticipate each other’s decision and then select decisions to further their individual goals. Nash equilibrium, where no one player would unilaterally deviate from one’s selected strategy considering the others’ strategies, is self-enforcing, so Nash equilibrium is rational, but may not be economically optimal overall [Hui et al., 2015].
The most fundamental one-shot noncooperative game is discussed first, where the levee system planning game is played only one time with simultaneous decision-making (an unusual case). Each player has multiple discrete levee height decisions as strategies, making its decision once, simultaneously, and irrevocably.

4.1. Identical Riverside Conditions
If each landholder only has two discrete planning choices of levee height (e.g., 1 and 3 m), Figure 3 shows the game’s normal payoff costs matrix. Each cell in the matrix contains a pair of payoffs (as costs, in this case) for a combination of heights chosen by the two landholders. In each cell, the number on the left is the payoff (cost) for rural Levee1 on the row, and the number on the right is the payoff for rural Levee2 on the column. Each player’s costs would depend on the levee height chosen by the other player, leading to changes in preferred decisions. In this example, Landholder1’s best levee height is 3 m for either of Landholder2’s choices, and Landholder2’s best levee height is 3 m for either of Landholder1’s choices. Thus, the strictly dominant levee height for both landholders is 3 m, which results in one Nash equilibrium. In this game, this equilibrium is also the system-wide economically optimal plan (having minimum total cost).

Landholders could have more discretized planning choices, for example, \(N=5\) planning choices of levee height from 1 to 5 m with 1 m increment. With a \(N \times N\) normal payoff matrix, we can find each landholder’s best heights responding to another’s choices (best response strategy), and identify any Nash equilibria and economically efficient outcomes.

A strategy that results in one landholder’s best payoff, given the others’ chosen strategies, is the best response (BR) strategy of one landholder to others [Fudenberg and Tirole, 1991; Gibbons, 1992]. Given the other landholder’s alternatives, each landholder’s best responses can be represented with a best responses curve. The points in Figure 4 each represent a landholder’s best response strategy. These points jointly create the best responses curve for one landholder, with 1 m (Figure 4a) and 0.1 m (Figure 4b) levee height increments. For instance, in Figure 4a, the best response strategy of Landholder2 is a 2 m high Levee2, given a 1 m high Levee1.

Based on Figure 4, the two identical riversides have the same best responses curves. If the levee height choice of one landholder exceeds a critical height (critical upper levee height \(H_c^1\) or \(H_c^2\)), the other landholder’s best response strategy becomes its individually optimal height \((H_c^1)\) or \((H_c^2)\) and stays constant further. Here the existence of a Nash equilibrium is affected by levee height increment (coarse or fine). A Nash equilibrium exists when \(\Delta H=1\) m (Figure 4a), both levee choices settle at 3 m high. No Nash equilibrium exists in this numerical example when \(\Delta H=0.1\) m (Figure 4b) or when \(\Delta H\) is even smaller, since the best responses curves cross over (no overlapping). With coarser levee height increments, where each landholder has fewer height choices, differences between the individual payoff costs of two neighboring heights are more likely to overlap to create a Nash equilibrium. The Nash equilibrium in this example with \(\Delta H=1\) m is between the maximum desired levee height (critical upper levee height \(H^*\)) and individual optimal levee height with smaller \(\Delta H=0.1\) m.

The best responses curves depend primarily on levee height and the resulting annualized construction cost. At low levee heights where annualized construction cost \(ACC\) is less than one’s individual optimal annual expected total cost \(TC(H^*)\), each player would always choose to build a levee higher than the other’s to avoid all remaining flood risk (which is then transferred to the other landholder). According to previous
assumptions, all residual flood risk goes to the levee with lower height (even by a single increment $\Delta H$).

Therefore, as a dominant strategy, each landholder would escalate its levee height higher than the other’s. There is a critical upper limit of one’s best levee height $H_c$ for each landholder (Figure 2). This critical upper levee height $H_c$ is where, with an additional height increment $\Delta H$, the resulted annualized construction cost $ACC(H_c + \Delta H)$ (all the flood risk is transferred to the other landholder) would exceed one’s individually minimized $TC(H_c)$. So it satisfies the condition that $ACC(H_c) < ACC(H_c + \Delta H)$, with $H_c > H^*$. The individually optimal $H^*$ becomes one landholder’s best response strategy if it would otherwise have to exceed the critical levee height. In this case with $\Delta H=0.1$ m, the individual critical levee height $H^*=3.9$ m and individually optimal levee height $H_c=3.1$ m are the same for the two landholders. So they stop increasing and reduce their levee heights to $H_c=3.1$ m when they have to exceed $H^*=3.9$ m to avoid the entire residual risk. (In practice, the cost of raising the levees to 3.9 m is irreversible, something examined later.)

4.2. Different Riverside Conditions

Figure 5 is the payoff costs matrix for each landholder having two discrete planning choices of levee height (e.g., 1 and 3 m) for different riverside conditions. The dominant strategies and Nash equilibrium in Figure 5 are similar to those in Figure 3. The strictly dominant planning levee height is 3 m for both landholders, leading to a Nash equilibrium that coincides with the system-wide economically efficient levee system plan.

Similarly, Figure 6 shows the best responses of each landholder given many more levee height choices, with 1 and 0.1 m levee height increments. A Nash equilibrium (3 m high Levee1 and 4 m high Levee2) exists when $\Delta H=1$ m, while no Nash equilibrium exists when $\Delta H=0.1$ m or with smaller levee height increments as there is no coincidence of the best response curves for the two landholders. The landholders’ best response curves have similar trends, but the individually optimal $H^*$ and critical $H_c$ differ for the two landholders. For $\Delta H=0.1$ m, $H^*_1=3.1$ m and $H^*_2=3.9$ m for rural Riverside1, and $H^*_2=3.7$ m and $H^*_2=4.7$ m for urban Riverside2. Similarly as a dominant strategy, each landholder still builds its levee slightly higher than the opposing riverside at low levee heights when annualized construction cost is less than $TC^*$, until reaching its $H_c$, and then a constant levee height response $H^*$ afterward.

In such one-shot noncooperative leved river system planning, given a range of planning choices and payoffs for each choice combination, there may be no Nash equilibrium (Figures 4b and 6b). Even if a Nash equilibrium exists, it may differ from the overall economically optimal leved river system planning (Figure 4).
6a). So the best choice may not be available for a rational player making desired levee height decisions from 0 to $H^c$, with a common $\Delta H=0.1$ m.

Such an institutional condition, with one-shot decision-making for leveed river system planning, while interesting, is rare, as the entire planning process is unlikely to be done in a short time without negotiations. Levee planning almost always includes extensive discussions and evaluations, followed by construction and maintenance, and levees can often be raised in the future, adding multiple shots in the future. One one-shot situation is emergency response during a flood event [Lund, 2012]. In a short time, two landholders must determine and implement their flood fighting actions, for example, sandbagging or ring levee construction as “levee planning.” Acting independently, a landholder would take its best flood fighting action regardless of the other’s decision.

5. Iterative Multiple-Shot Noncooperative Planning With Reversible Decisions

With the institutional arrangement of multiple discussions on the leveed river system planning game, we extend the one-shot noncooperative planning to multiple shots. And first, we let the decision-making be reversible. A multiple-shot game, also known as a “continuous game” [Madani and Hipel, 2012], would include multiple opportunities for a player to choose its best alternative during the course of the game. A player also can account for the implications of other players’ current and future actions. The number of shots (steps) that the planning game is played here occurs when cycling or convergence occurs.

In multiple-shot leveed river system planning, noncooperative landholders could alter their individually best levee height responses given the others’ decisions in subsequent periods [Hui et al., 2015]. Assuming recurring leveed river system planning decisions, landholders could bargain over their levee heights multiple times, until they reach converged heights or the allowed bargaining time ends. At each iteration, one or
both landholders would state its best levee height. Decision-making in the game can be leader follower, where landholders make choices one by one, or simultaneous, with landholders making choices at the same time. Each landholder decides its best individual height starting from a 0 m high levee. Initial planning decisions and initial leader(s) of the game may affect the final results. Because the results of leader follower and simultaneous games are similar in this case [Hui, 2014], only the results with simultaneous decisions are presented here.

Where two riversides are identical, the best response of each landholder’s levee height to the other’s decision is identical in successive steps. Figure 7a presents the best responses curves of multiple-shot leveed river system planning with reversible decisions that are made simultaneously, in 75 steps and with $\Delta H = 0.1$ m height increments. The best individual levee heights do not overlap, and no Nash equilibrium exists in this game. Best individual levee heights for the two landholders are trapped following the same cycles: from $H_c^1 = 1$ m to $H_c^2 = 3.9$ m (reversibility allows each levee owner to reduce construction cost by lowering levee height). As landholders increase their best individual heights responding to one another in iterative steps, one (or both) landholder’s best individual height approaches the critical upper limit of its best levee height $H^*$. Once a landholder proposes a levee higher than $H^*$ to avoid flood risk, the best response strategy is reducing its levee height back to $H_c^1$ with an individually minimized $TC^*$, where the cycle of levee escalation repeats. This form of leveed river system planning can be interminable. Nonconvergence of the best levee heights is because of the discontinuous individual annual expected total cost, depending on the opposing levee decision.

With different riverside conditions, rural Riverside1 with smaller potential damage has a smaller $H_c^1$ and $H_c^2$. The best individual heights in the successive steps are similar for the two landholders. Similar to Figure 7a, Figure 7b presents the best responses curves for such a game, where the landholders’ best responses of height decisions also get stuck in cycles. But a slight difference is that urban Landholder2 will follow rural Landholder1’s trapped cycle, one step behind. Rural Landholder1 with smaller potential damage and smaller $H_c$, would first reduce the best individual height to $H_c^1 = 3.1$ m when Levee1 must exceed $H_c^1 = 3.9$ m to avoid the entire residual risk. Urban Landholder2 with a higher $H_c^2$ would follow Landholder1’s cycling decisions one step later due to the game’s structure. There is still nonconvergence and no equilibrium in this game.

Overall, no convergence and no equilibrium exist in such iterative multiple-shot levee system planning games where decisions of best individual heights are reversible, for identical or different riverside conditions.
conditions. The best strategy is not available for each landholder. Possible levee heights are between zero and $H^*$, and more likely within the cycle region from $H^*$ to $H^*$.

In reality, this iterative multiple-shot planning game with irreversible decisions would be highly unusual, except for noncooporative discussions in river system levee planning (when proposed levees can be lowered for cost savings). For most leveed river system problems, levees are unlikely to be lowered after construction. More typical situations are constructing new levees and upgrading existing levees, where landholders have irreversible decisions that cannot recoup construction costs by lowering their levee heights.

### 6. Iterative Multiple-Shot Noncooperative Planning With Irreversible Decisions

Where decisions are irreversible, a landholder in iterative multiple-shot levee system planning can only increase or stay at the same levee height. Irreversible multiple-shot noncooperative games are observed and studied in many areas, such as policies, environment, fishing, and ecosystem protection [Carraro and Siniscalco, 1993; David, 1994; Sumaila, 1999; Madani and Zarezadeh, 2014]. Such irreversible iterative multiple-shot levee system planning can easily lead to economically inefficient or even damaging outcomes, if independent and self-interested landholders are shortsighted for the future.

In California, levees have been used to manage flooding since the mid-1800s with inefficient escalating levee construction in the early era [Kelley, 1989; Hanak et al., 2011]. From 1867 to 1880, levee districts located along the Sacramento River raced one another to build higher levees on each riverside [Russo, 2010]. During that period, when landholders along the Sacramento River and its tributaries could have collaborated on flood control projects (in theory), flood-prone landholders escalated their levees to force floodwater onto their neighbors, since channeled floodwater would overflow or breach smaller and weaker levees. The resulting escalation of levees in the Sacramento Valley became ineffective and economically inefficient, and ultimately led to violence against some neighboring levees since it became less expensive to demolish the opposing levee than to strengthen one’s own levee [Kelley, 1989].

Results and discussions below are for irreversible iterative multiple-shot river levee system planning. Still each landholder decides its best individual heights starting from a 0 m high levee. Only simultaneous decision results are presented for illustration.

#### 6.1. Identical Riverside Conditions

Given identical riverside conditions, Figure 8 presents results for irreversible multiple-shot leveed river system planning with simultaneous decision-making. Levee height increments are $\Delta H = 0.1$ m for 100 steps in Figure 8a and $\Delta H = 0.05$ m for 200 steps in Figure 8b. Height increments of 1 m and 0.01 m were also examined, but not plotted here as the general conclusions are similar. For identical riversides, the best individual heights of the landholders converge at the same level ($H'_1 = H'_2$) in both Figures 8a and 8b. However, the converged heights vary with the height increment $\Delta H$. Neither converged height is the system-wide optimum height of 2.7 m. And for the same $\Delta H = 0.1$ m, the converged heights in Figure 8a (4.7 m) exceeds the maximum desired levee height (critical upper levee height 3.9 m) as shown in Figure 8b.

For each landholder given irreversible decisions with nondecreasing levee heights, the best individual levee height is determined by the added ACC and the reduced EAD with an additional $\Delta H$.

At step $t$ when each landholder makes its move, both landholders have their best individual heights $H_{1,t}$ and $H_{2,t}$ that transfer all residual flood risk to the opposing landholder considering the other’s previous best individual height. So their best individual annual expected total costs only include annualized construction costs, for example, for Landholder 1:

$$TC_{1,t}(H_{1,t}, H_{2,t-1}) = ACC_{1,t}(H_{1,t})$$

At step $t+1$, each landholder’s levee height decision can be the same as previous step $t$ (e.g., $H_{1,t+1} = H_{1,t}$), or increase to $\Delta H$ higher than the other’s levee at step $t$ (e.g., $H_{1,t+1} = H_{2,t} + \Delta H$). For example, for Landholder 1, if $H_{1,t+1} = H_{1,t}$, the landholder would absorb all flood risk since Landholder 2 is likely to increase its levee height. Where both landholders keep at identical levee heights, they equally share the flood risk (unless both levees would fail simultaneously). If $H_{1,t+1} = H_{2,t} + \Delta H = H_{1,t} + \Delta H$ (multiple $\Delta H$ increases are not...
If \( H^{t}\) is the converged levee height of Landholder1 (\( H^{t}\)), \( H_{1,t+1}=H^{t}+\Delta H \) will cost less than \( H_{1,t+1}=H_{1,t}+\Delta H \) and Landholder1 would stay at its best individual height \( H^{t} \), in all following steps. Once the increased ACC (\( ACC_{1}(H_{1,t}+\Delta H)−ACC_{1}(H^{t})\)) exceeds the reduced EAD (\( EAD_{1}(H^{t}) \)) for an additional \( \Delta H \), a landholder would not raise the best individual height any more. So the convergence condition for \( H_{1,t} \) is that:

\[
EAD_{1}(H^{t}) \leq ACC_{1}(H_{1,t}+\Delta H)−ACC_{1}(H^{t}) \tag{9}
\]

where \( EAD_{1}(H^{t})=DP_{1}×\left[1−F_{0}(Q_{1}(H^{t}))\right] \) and \( ACC_{1}(H_{1,t}+\Delta H)−ACC_{1}(H^{t})=\left[s×\left[L×\frac{8c}{\Delta H}×\frac{1}{1+\frac{1}{\Delta H}}\right]×\left(\Delta H^{2}+2H_{1,t}×\Delta H\right)×\frac{1}{1+\frac{1}{\Delta H}}×\frac{1}{1+\frac{1}{\Delta H}}\right] \).

Therefore, a smaller \( \Delta H \) leads to larger converged best individual heights. And the converged heights could be greater than the critical upper levee height in the one-shot noncooperative game, due to irreversible decision-making. This can be shown mathematically by substituting the formula of \( EAD \) and \( ACC \) from equations (5) and (6) to equation (9). Theoretically, without any limitations from levee design standards, the converged best individual levee heights could be infinitely large if the levee height increment \( \Delta H \) is infinitely small. But such a situation rarely exists for practical reasons, such as substantial fixed costs (engineering, site preparation, etc.) for any levee work, design standards for levee height increments, financial limits, and upper limits on levee height, particularly where \( \Delta H \) needs to be physically defined and cannot become infinitely small in practice.

### 6.2. Different Riverside Conditions

Similar to Figure 8, Figure 9 shows irreversible multiple-shot noncooperative river levee system planning for differing riverside conditions, with simultaneous decision-making. Levee height increments are \( \Delta H=0.1 \) m in Figure 9a, and \( \Delta H=0.05 \) m in Figure 9b. Compared to Figure 8, best individual heights of two
landholders converge after similar steps in Figure 9. However, best individual height convergence differs between the two differing landholders ($H^*_1 \neq H^*_2$). For $\Delta H = 0.1\ m$ or $\Delta H = 0.05\ m$, the best individual height convergence of Landholder2 is greater than that of Landholder1 by one $\Delta H$ ($H^*_2 - H^*_1 = \Delta H$). The impacts on the variance of best individual height convergence from levee height increments are similar to those discussed for Figure 8. In neither case do the best individual levee heights converge on the optimal set of levee heights ($H^*_1 = 2.7\ m$, $H^*_2 = 2.8$ or $2.75\ m$).

For both landholders, one’s best individual height convergence $H^*$ is inferior to the individually optimal $H^*$ that corresponds to a smaller $TC^*$. A landholder that is nonmyopic, could avoid the cost of such decision-making, which is the individual total cost difference between with the best individual height convergence $TC^*(H^*)$ and with the individually optimal height $TC^*(H^*) - TC^*(H^*)$. Therefore, keeping one’s best individual height low and stop increasing at early steps could avoid much higher cost later. Nonmyopic landholders may be willing to take such a “strategic loss” [Madani, 2011].

7. Discussions and Limitations

Table 1 summarizes different institutional and flood damage cases examined in this study and their major results. Each section addresses a planning case in this table.

The system-wide economically optimal river levee plan would have the minimum overall net expected flood damage and infrastructure costs. However, without interference (e.g., authority or compensation) to incentive or force collaboration between independent landholders, economically inefficient plans are the likely outcome under noncooperation: no Nash equilibrium of best strategy or inferior equilibrium of converged heights. Only a nonmyopic landholder who can foresee the inferior ending results may take the “strategic loss” [Madani, 2011] by choosing an individually optimal levee planning that would at least give better results than noncooperation. The myopia causes extra costs and inefficiencies in levee system, happening historically when rational players made decisions, such as the above mentioned levee battles in California [Kelley, 1989]. Stable solutions differ from optimal solutions. Once a system-wide decision-maker/regulator understands the effect of myopia on inefficient planning, it can consider designing mechanisms to create an overlap between stability and optimality [Read et al., 2014].

A cooperative game can determine how to allocate the benefit from improving the overall cost of transforming a noncooperative game into cooperative system-wide plan, where appropriate compensation or...
authority can incentivize or force collaboration among players. Many studies discuss the core of a cooperative game, where the collaborative payoffs are preferable [Nash, 1953; Aumann and Dreze, 1974; Curiel, 1997; Giglio and Wrightington, 1972]. Further studies can examine the core of this leveed river system planning game to help planning processes and institutions achieve a more economically efficient levee system. Particularly for similar games involving multiple players, for example, a ring levee system with many sections owned separately, future studies may prove that no coalition has a value greater than the collaborative payoffs and no coalition has an incentive to leave the grand coalition [Aumann and Maschler, 1961; Madani and Dinar, 2012a].

The game theoretical analysis in this study is limited by the following major assumptions and simplifications. To apply such analysis to similar problems, these limitations should be considered and properly addressed.

1. Potential system failure here is driven only by a levee height increment. Reality is often more complicated. Two riversides may encounter different risk distributions owing to additional levee failure modes (through-seepage, underseepage, etc.), in addition to overtopping [Wolff, 1997; Cenderelli, 2000; Foster et al., 2000]. This would more evenly, and less predictably, distribute residual flooding risk between the two river banks, and would likely alter game results. Additional levee failure modes should be included in analyzing system failure, where levee structure and failure have more complete physical representations.

2. Flood risk may be calculated differently than the expected (residual) damage cost, which determines the payoff function for each landholder, and therefore the game results. Intuitive risk of each landholder and the expected economic payoff function may differ; a risk-averse landholder would be less willing to take flood risk than the expected value would indicate. Risk aversion of landholders should be considered in flood risk estimation and in distinguishing identical or different riverside conditions.

3. Levee system planning should consider levee length effects, which is ignored in this study. Levee length affects construction cost directly. It also affects expected flood damage cost, as a longer levee has higher chance of failure. This is omitted from the flood risk calculation here.

4. The examined levee system planning game is static, while the game structure is potentially evolving over a long time, owing to changes in land use and climate [Zhu et al., 2007].

5. Hydraulic and hydrologic uncertainties exist from Manning’s equation and commonly assumed stationary probability distribution of flood flow. Where additional information is available, these uncertainties should be considered.

8. Conclusions

We apply noncooperative game theory to examine decision-making with risk-based analysis for simple leveed river system planning with two independent and self-interested landholders on opposite riversides who build their own levees, with the same or differing riverside conditions. The overall economically optimal plan for the levee system minimizes annual expected total flood costs system-wide. However, economically efficient leveed river system plans are unlikely from independent and self-interested landholders on
opposite riversides optimizing best individual levee heights separately in a noncooperative setting. The result has similarities to the general problem of managing common goods—the so-called tragedy of commons [Gordon, 1954; Hardin, 1968]. The noncooperative games often do not result in a system-wide optimal solution: a rational landholder may have no best strategy, get stuck in cycling decisions or accept inferior best individual heights. Iterative multiple-shot games show problems of nonconvergence and no equilibria (where decisions are reversible), and show a cost of decision-making myopia for the future (where decisions are irreversible), in which case “strategic loss” might become desirable to prevent higher costs later if landholders are nonmyopic. Higher-level decisions and/or compensation for transferred flood risk (which introduces cooperation or authority) are often needed for economically efficient levee system plans [Eijgenraam et al., 2014].

By analyzing different types of games, from the fundamental one-shot noncooperative game to more realistic multiple-shot noncooperative games with irreversible decisions, practical institutional arrangements can be explored. With sufficient examination of games representing different institutional arrangements, a system-wide decision-maker, funder, risk-sharing arrangement, or regulator, could determine how to lead and organize leveed river system planning to reach better overall outcomes.

The applicability of a game theoretical framework can be increasingly limited when more assumptions and simplifications are needed. But some general conclusions from game theoretical analysis seem unlikely to change: noncooperation often will result in overinvestment in levee protection, overall individuals could benefit more from cooperative solutions (supported by authority or compensation), and “strategic loss” [Madani, 2013] can help an individual avoid some worst outcomes in noncooperation situations.

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