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STORAGE TO BE PROVIDED IN IMPOUNDING RESERVOIRS FOR MUNICIPAL WATER SUPPLY*

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WITH DISCUSSION BY MESSRS. HIRAM F. MILLS, T.U. TAYLOR, F. B. MARSH, L. J. LE Conte, E. P. Goodrich, JAMES L. TIGHE, H. T. CORY, AND ALLEN HAZEN.

There is undoubtedly a definite relation between the storage provided in an impounding reservoir "on any stream and the quantity of water which can be supplied continuously by it. The relation, however, is a complex one; and our knowledge of its character is limited. The following study is made to see how far it may be possible to separate this complex relation into parts, some of them being of such a nature that they may be studied separately with definite results, and afterward to treat all the remaining variation on the basis of probabilities, using all data from a number of different streams; -and to study them in comparison with the normal law of error.

Among the elements that can be studied separately are the following:

1. The size of the catchment area.
2. The mean annual run-off per square mile.
3. The portion of water area and the loss by evaporation' from it.

This relation is a complex one, and data for determining it are less adequate than could be desired. Nevertheless, some approximations can be reached.

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4. The natural storage in lakes, or in deposits of sand and gravel and other pervious materials. Only approximate results for natural storage can be reached, but as these are found to have a great influence on the required storage, especially at relatively low rates of draft, they must be considered.

5. Regularity in annual flow. Some streams have comparatively regular flows; in others the variation is much greater. This difference in regularity of flow can be taken into account by finding a coefficient determined from the record of each stream, and bringing this into the statement in such a way that variations from the normal are stated in terms of the "standard variation". In this way, records of streams having more regular flows and those having less regular flows may be compared with reference to other matters.

For the present, all remaining elements of variations of flow of every description will be thrown into one group and studied in connection with the normal law of error.

In general, it may be said that, as more information is secured as to any part of the whole problem, it tends to reduce unexplained variations, and to permit more accurate analysis of their parts. The whole study, therefore, becomes one of successive approximation. The results herein. contained are to be taken as only one step in the development. It is to be expected that further study will show reasons for deviations, the causes of which are not now apparent, and will ultimately lead to more certain and accurate knowledge of the whole subject.

The method is developed in connection with the study of the normal law of error are well suited to an investigation of this kind. There is a presumption that they may be applied, growing out of experience with other kinds of data, but the presumption is not so strong that it is to be accepted without a careful study of the best flow data available, to see how close the agreement really is, and how far it may be depended on.

The first requisite to a successful study of probabilities is to have ample data. There are only a few cases where the records of stream flow cover a longer period than 25 years, and the longest record here used, that of the Croton River, covers only 45 years. No one of these records, taken by itself, is sufficient to serve as inadequate

basis for forming judgment of the probability of the recurrence of conditions which are only to be expected at intervals longer than are covered by these records.

In order to form a. judgment of such conditions, we may either resort to the normal law of error and assume that, if it applies reasonably to the data for short periods, it will also apply to longer ones ; or we may combine the records of several streams, after eliminating the elements that can be separated. and. eliminated, thus forming a single series containing the elements" of unexplained variation reduced as nearly as possible to a common basis. In this way a series of results is built up which may be taken as representing the unexplained variations in the flow of a single stream for hundreds of years.

Such an artificial record is open to some objections. All the records of which it is composed were obtained in the same general period of time, and any changes in climatic conditions that might take place in a long term of years are not reflected by it. Nevertheless, such an artificial series seems to be the best means now available for finding approximately some of the relations between flow and storage.

Study Divided into Two Parts.-An impounding reservoir serves two purposes. These run into each other more or less, and overlap, but they are nevertheless reasonably distinct, and can be best considered separately.

The first is to balance the fluctuations in flow during the seasons of one year. That is, to hold the flood flows of the winter and spring and make the available for maintaining the service during dry periods in the following summer and fall. This will be called the monthly storage.

The second is to hold the surplus water of wet years and make available in the dryer years that follow. This will be called the annual storage.

These matters will be taken up for discussion separately, and afterward the results will be combined in a single statement.

Approximate Methods Used.-,ln all hydraulic data the probable error of measurement is considerable. There is, therefore, no justification for the application of extreme refinements in methods of

calculation. With this in mind, slide-rule calculations have been used. In most cases, available record are given as monthly averages, and each month has been taken as one-twelfth of the year, regardless of the number of days it actually contained. In a few cases, daily or weekly records are available, and a brief investigation has been made as to the probable error involved by the use of monthly average figures instead of daily ones. Some changes of method have been made during the course of the work, and minor discrepancies resulting therefrom (too small to be significant) have not always been corrected. Where records from several streams are to be averaged, a weighted average is used in which each is given a value in proportion to the length of the records from which it is obtained.

Land Area Only as Basis.-In the case of many of the streams included in the following study, all or nearly all the catchment area is land; that is to say, it is not occupied by the water surface of reservoirs. In other cases, reservoirs or lakes have occupied a certain percentage of the area, and this percentage has gradually increased during the period covered by some of the records. The published records of flow are based on the whole area, including water surface. For the purpose of this study, the method of taking only land area as. a starting point, as suggested by Frederic P. Stearns, Past-President, Am. Soc. 0. E., has. been followed. The published figures are revised by dividing the run-off per square mile from the total area, by one less the proportion of water area.

DATA USED

The first step is to reduce existing data to a land-area basis, and make a tabular statement showing the average flow per square mile for each catchment area for each month for the whole period covered by the observations. The records of flow of the following streams have been used:

Sudbury River. This is a part of the Boston water supply, with a catchment area of 75 sq. miles. The records cover the period from 1875 to 1896, inclusive, during which time the water area ranged from *2* to 4 per cent. The more recent records of the Sudbury River are not used, because, beginning with 1897, the water for Boston from the Wachusett catchment area was drawn through the Sudbury system, and the Sudbury figures are obtained as the difference between

the measurements of the water entering and leaving. *As*·the quantities passing through are much larger than those originating in the area, the measurement is not believed to be- sufficiently accurate to justify the use of the records since this condition has existed. The Sudbury catchment area is rolling, inhabited, and cultivated, and has much sand and gravel.

Wachusett Reservoir (South Fork of the Nashua River). This is also a part of the Boston water supply. The records, cover the period from 1897 to 1911, inclusive. The catchment area of 118 sq. miles is somewhat more hilly and higher in elevation than the Sudbury. It also. contains a large quantity of sand and gravel. The water surface has ranged from 2 to 7 per cent.

Groton River.-This is a part of the New York water supply, with .a catchment area of 339 sq. miles at the Old Croton Dam increased in 1906 to 360 sq. miles at the New Croton Dam the records cover the period from 1868 to 1912, inclusive, during _which time the water area has ranged from 2 to 5 per cent. The catchment area is rolling, is cultivated to a considerable extent, and has a large quantity of sand and gravel.

Manhan River.-This has a small catchment area of 13 sq. miles, forming part of the water supply of Holyoke, Mass. The measurements were made by weirs, with unusual care, and cover the period from 1897 to 1910, inclusive. The area is rolling, and the quantity of sand and gravel is large.

Catskill Btreams.-These include the Esopus, 378 sq. miles; Schoharie, 240 sq. miles; and Rondout Creek, 105 sq. miles. The gaugings were made by the City of New York, and cover a relatively short period. The catchment areas are steep and mountainous and largely wooded. The soil is generally impervious, and. there is little sand and gravel. The records are not of sufficient length to cover the dryest periods, but they are useful as indicating the ordinary conditions of storage required in catchment areas differing radically. in physical characteristics from those previously mentioned.

Pequannock River.-This furnishes the water supply for Newark, N. J.· It has a catchment area of 62 sq. miles of hilly, almost mountainous country, cultivated to a moderate. Extent, but largely covered with second-growth forest. The record covers the period from

1892 to 1911, inclusive, and is made up of Venturi measurements of the water taken out of the catchment area for use, and weir measurements of the water washing over the intake dam. The water area is about 4 percent.

Philadelphia Streams. Three streams, Perkiomen Creek, 152 - sq. miles, Neshaminy Creek, 139 sq miles, and Tohickon Creek,102 sq. miles, were proposed many years ago as sources of additional water supply for Philadelphia. They were not used as proposed, but careful gaugings extending over a period of 25 years, are available. The country is hilly and partly cultivated.

Gunpowder River.- This furnishes the water supply for Baltimore, Md. The catchment area is 308 miles of hilly, rolling country, under a good state of cultivation. The hills are high and steep, but there is a deep cover of fine-grained micaceous sand material on a large part of the area, and this serves to store a large quantity of water, so that the ground-water flow if this stream is larger relatively than that of any other stream considered. The records are made up of the quantity of water drawn for use by the City of Baltimore, and that flowing over the intake dam, calculated from the records of gauge heights covering the period from 1883-1911, inclusive. In considering these records, in their report on the water supply Messrs. Freeman and Stearns reduced them slightly because they believed that the coefficient to be used in the weir formula, for the crest of the dam as it existed, was 1ower than had been assumed in making up the quantities used by the Water Department. In this calculation the records are used without correction.

Merrimac River.-The Merrimac River drains an area of 4 634 sq. miles. The record of the flow at the Lawrence Dam has been kept by the Essex Company. Lake Winnepesaukee and many smaller lakes, comprising 2.6% of the total catchment area, are included in the area, and storage in them, either natural or with the aid of the control works at the outlets of some of the lakes is an element in maintaining the regularity of the flows. No correction –has been made in the Merrimac flows,

Hudson River.-A record of the flow has been kept at Mechanicsville, where there are numerous lakes on the catchment area, and natural storage is an element in maintaining the flows. No correction has been applied for this area.

Colorado Streams.-The gaugings of three streams used by the Denver Union Water Company are available

1. Bear Creek ; record, 1900 - to 1911, inclusive, and a catchment area of 172 sq. miles ;
2. South Fork of the South Platte River; record, 1900 to 1911, inclusive, and a catchment area of 1 796 sq. miles
3. South Platte River at Platte Canyon; record, 1903 to·1911, inclusive, and a catchment area of 2 688 sq. miles.

These streams were included as typical ones in a dry country where the variation in flow from year to year is much greater than in any of those previously mentioned. The seasonal fluctuation in flow also depends on different climatic conditions, especially on the accumulation of snow in the high mountains, and its melting in summer. These streams were included in the study in order to see how far the same methods of calculation would be applicable to them, and with the idea that they would be included in summaries with the·eastern streams only if it became apparent that doing so would not modify materially the standards reached. Including these western streams has the important advantage that it tends to broaden the methods used for comparing data, so that then are applicable under a greater variety of conditions.

Not all the sources mentioned were used for all parts of the following study, but only such records as seemed most appropriate in relation to each point taken up in turn.

Additional Records. – It would be possible, by using the records of the United States Geological Survey, greatly to extend the investigation. In the first study it is thought best to use only a limited number of selected data. It is by no means an easy matter to gauge streams accurately, especially under winter conditions, in northern climates, with large quantities of ice in the water. It is necessary, therefore, to use these records with caution, because such errors may exist in some of them. Long-term records are obviously more useful than those of short term, and the latter have been considered only in the case of a few streams representing types of catchment area not otherwise included.

Calculation of Storage.-Calculations were made for each year for each of the streams investigated in order to determine the storage required to maintain each of a number of assumed rates of draft. For most of the streams, for which the flows were given in gallons per square mile, these calculations were made £or rates of draft of 100 000, 200 000, 4:00 000, 600 000, 800 000, and 1000 000 gal. daily per sq. mile 0£ land area, or £or so many of them as required storage and could be supported. In the case of streams where the records were in cubic feet per second, rates of draft were taken giving about the same general range.

Basis of Stating Storage.-There are several ways in which the storage may be stated: In relation to the land area, in terms of the mean annual run-off, or in terms of the proposed maintainable draft. When storage is based on the tributary area, it may be stated as inches of run-off or millions of gallons per square mile of tributary area. These and similar terms bear fixed relations to each other. Thus 1 in. of run-off is always equal to 17 400 000 gal. per sq. mile.

When these units are used, it should be stated whether they relate to the whole area or to the land area only, as either form may be used. When the storage is based on the mean annual flow the percentage of it required to £11 the reservoir from bottom to top is usually given. When the storage is based on the maintainable yield it is most conveniently stated in days' supply. Thus, if a given reservoir holds 100 000 000 gal., and is used for an output of 2 000 000 gal. per day, 50 days' storage is said to be provided. Each of these three ways of stating storage has advantages for certain purposes, and they are all used in this discussion.

MONTHLY STORAGE.

Each Year by itself.- It is assumed, in the first part Patt of this study, that the reservoir is full each year at the beginning of the summer dry period, and no account is taken of any deficiency that might exist from operations of previous years. The figures thus reached are referred to as those for “each year by itself" or as "monthly storage". A second set of figures is made in which the deficiency at the beginning of any year is carried forward. The figures thus reached are spoken of as "cumulative". The cumulative figures are comparable with those made by Messrs. Stearns, FitzGerald, and Freeman for a zero water area.

i.

At the outset it was supposed that the cumulative figures would be the most important and those representing each year by itself were csrr~ed as a convenient and perhaps useful check: It turned out, however, that the figures for each year by itself could be reduced to more definite and satisfactory order than was possible for the cumulative figuress, and in the final study no use is made of the latter, except as a check on the results otherwise reached.

The monthly· storage figures, representing the storage for each .ye .. by. it.. sel:£, are used as a conveni.e nt st~p in the. . p.r ocess of development, ~ . end do not always represent the '~hole quantity of storage required; ss they contain no allowance for the annual storage necessary to hold ·the ex~ water of wet years and make it available in dry ones.

The·figures for monthly and annual fiow for each of the streams, and the computed storage therefrom for several assumed rates. of draft, are. compiled in Table 1. As the original of this table is very long, and as all the fgures used in the subsequent discussion are shown graphically in the diagrams which follow, the table is not presented in full. but only one part of it, containing the records of one stream, in order to show its form.

Plotting Results.-On Fig. 1 are plotted these results for the Wachusett Reservoir. The quantities of storage for each rate of draft are arranged in order of magnitude and plotted on lines equally spaced on the diagram. These lines are not reproduced, but the points plotted show their position. Lines showing the percentage of the total number of years are added as a more convenient basis for further discussion. Lines connecting them approximate in shape to the letter "S", the middle part of the curve being moderately straight, and the curvature near the ends much sharper.

Smooth curves have been drawn to show the most pro~able curve in case the normal law of error applies to the data. The question as to whether it does apply will not be discussed at this time, but the lines drawn with its aid give a better idea of the normal shape of the cu.rves than can be obtained from the few points that constitute this particular series. I£ the data were much more numerous, a fair approximation of positions near the ends, that is to . say, of the probable storage required to maintain drafts in years so dry that they recur only once in 20 or 50 years, could be obtained.

TABLE 1.-(in part)-Average run-off, by months, in thousands of gallons per day per square mile of net land area, for Wachusett Reservoir.

Year.	Percentage of water area.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1897	2.2	815	954	2 822	1 669	1 189	1 208	1 474	916	389	248	1 312	2 326	1 281
1898	2.2	1 598	1 672	3 157	2 072	1 421	847	341	1 355	692	1 543	2 219	2 107	1 586
1899	2.2	2 139	1 115	2 838	3 452	882	574	362	242	256	251	440	367	1 075
1900	2.2	814	4 144	3 806	1 616	1 413	591	222	201	131	289	895	1 605	1 292
1901	2.2	531	364	2 779	5 098	2 790	1 007	488	524	327	662	529	3 307	1 541
1902	2.2	1 714	1 432	4 081	2 207	1 054	419	299	304	247	973	650	1 889	1 276
1903	2.4	1 296	2 185	3 507	2 293	583	2 183	639	486	384	705	649	976	1 316
1904	3.6	684	962	3 120	3 095	1 554	791	515	368	512	360	356	457	1 063
1905	4.1	1 320	472	3 132	1 686	464	565	381	335	1 280	383	461	1 061	965
1906	5.1	1 193	1 082	1 960	2 222	1 615	1 248	768	623	292	559	790	836	1 099
1907	6.0	1 551	736	1 805	1 528	1 027	822	356	93	861	1 470	2 702	2 086	1 255
1908	7.0	1 869	1 867	2 357	1 364	1 521	484	237	476	95	170	134	416	910
1909	7.0	637	2 748	2 289	2 604	1 303	680	251	208	224	97	391	578	987
1910	7.0	1 985	1 984	2 839	1 112	655	886	67	200	156	73	381	420	890
1911	7.0	832	672	1 440	1 498	496	377	61	202	195	773	1 113	1 147	733

Computed storage, in millions of gallons per square mile of net land area, for several rates of draft. Each year is considered by itself, except that where cumulative storage is greater, the figure for it is given in parentheses.

Year.	Computed Storage, in Millions of Gallons per Square Mile of Land Area, for Drafts as Marked.					
	100 000	200 000	400 000	600 000	800 000	1 000 000
1897.....	0	0	5	17	29	44
1898.....	0	0	2	8	14	25
1899.....	0	0	15	52	95	147
1900.....	0	2	23	48	78	127
1901.....	0	0	2	14	45	75
1902.....	0	0	11	35	59	95
1903.....	0	0	0.5	10	35	77
1904.....	0	0	3	31	68	118
1905.....	0	0	3	20	53 (127)	95
1906.....	0	0	3	11	30 (189)	65
1907.....	0.2	3	11	23	35 (96)	57
1908.....	0.2	6	27	68	116	165
1909.....	0.1	3	25	57	97 (154)	140
1910.....	1.0	9	34	70	112 (206)	173
1911.....	1.2	4	23 (59)	51 (153)	82 (297)	119

With data. no more numerous than those in the record shown, by Fig. 1, the method is a clumsy one and cannot be expected to yield close results: It may be pointed out, however, that the method

which has been most commonly used for estimating storage has been that of considering only the highest trerm in the storage series, and that no consideration has usually been given to the other terms. Under these conditions, the degree of dryness of the year that controls is a matter of chance. The longer the record period, the greater is the chance that it will include a very dry year. Plotting even this crude way is an improvement, in that it gives some idea of the range of results and the frequency of recurrence of extreme values.

Probability Paper.-The practical difficulty with the plotting on Fig. 1 is the great curvature of the lines showing the required storage. This difficulty is so great as to make the method unsatisfactory in most cases; but it has been removed by using paper ruled with lines spaced in accordance with a probability curve, or, as it is otherwise called, the normal law of error. The spacing of the lines for this paper was computed from figures taken from probability curve tables, and arranged so that the line which represents the summation of the probability curve, when plotted on it, is straight. If the data for any series correspond strictly with the normal law of error, the points plotted on this paper will all be in a straight line. If the data approximate the normal law of error, the line through the points will approximate a straight line. Even though the deviation from the normal law of error is considerable, a line with only a moderate curvature may represent it fairly well.

In plotting observations on this paper, either o£ two methods may be used. First, all the results may be divided into classes between certain limits, and the corresponding limits in the other direction may be determined and plotted. This the better method where the number of terms in the series is large. It cannot be well used with the relatively small number of terms ordinarily constituting a series of storage data. In the second method the whole space is divided into as many vertical strips as there are terms, and the figure for each term, arranged in its order of magnitude is plotted at the percentage which corresponds to the middle of its strip. That is to say if there are 50 terms in the series, each is taken to represent 2% of the whole space. The first term will be plotted at the middle of the first 2% strip; that is, on the 1% line; the second term will be plotted in the middle of the second strip or on the 3% line, etc.

The position for plotting results can be obtained with sufficient accuracy with a 10-in slide rule. The decimal position of the *m*th term in the series of *n* terms is found to be P = (2m-1)/(2 n).

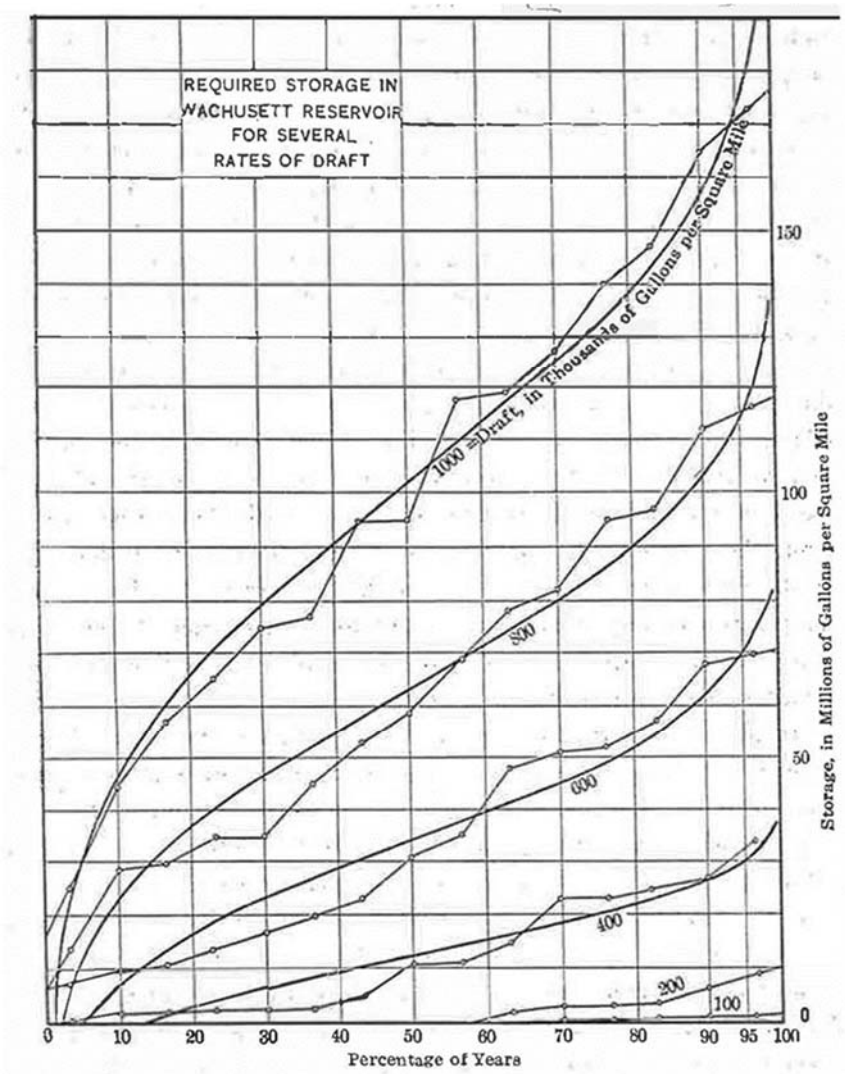


FIG. 1.

Storage Data on Probability Paper.--On Fig. 2 are plotted on probability paper the same data that were plotted to natural scale in Fig. 1. It is seen that the sharp curvature at the ends is entirely eliminated. The lines representing the several series have only a

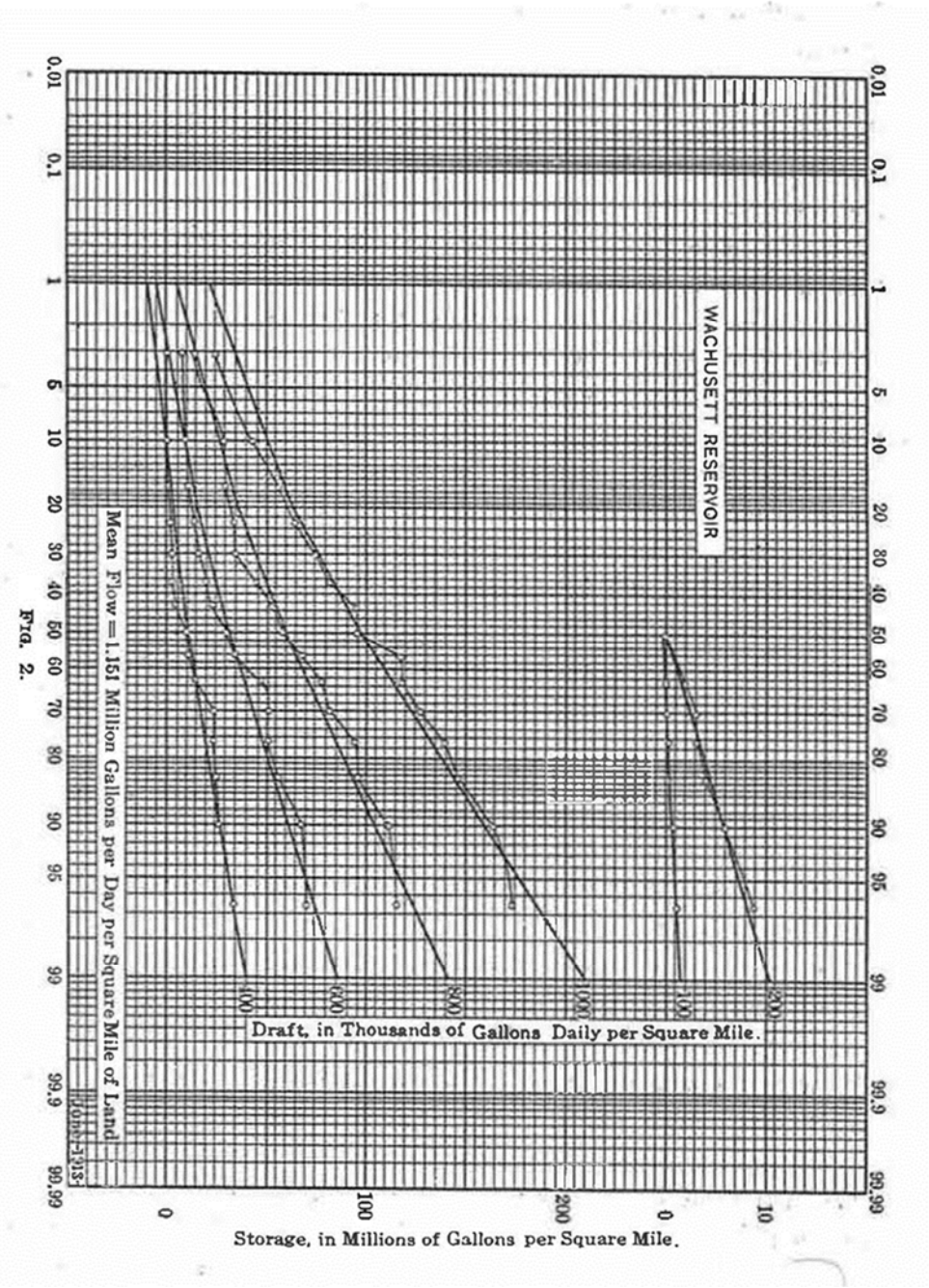


FIG. 2.

moderate curvature, and this is much the same in the different cases. This method represents the data more satisfactorily than arithmetical plotting.

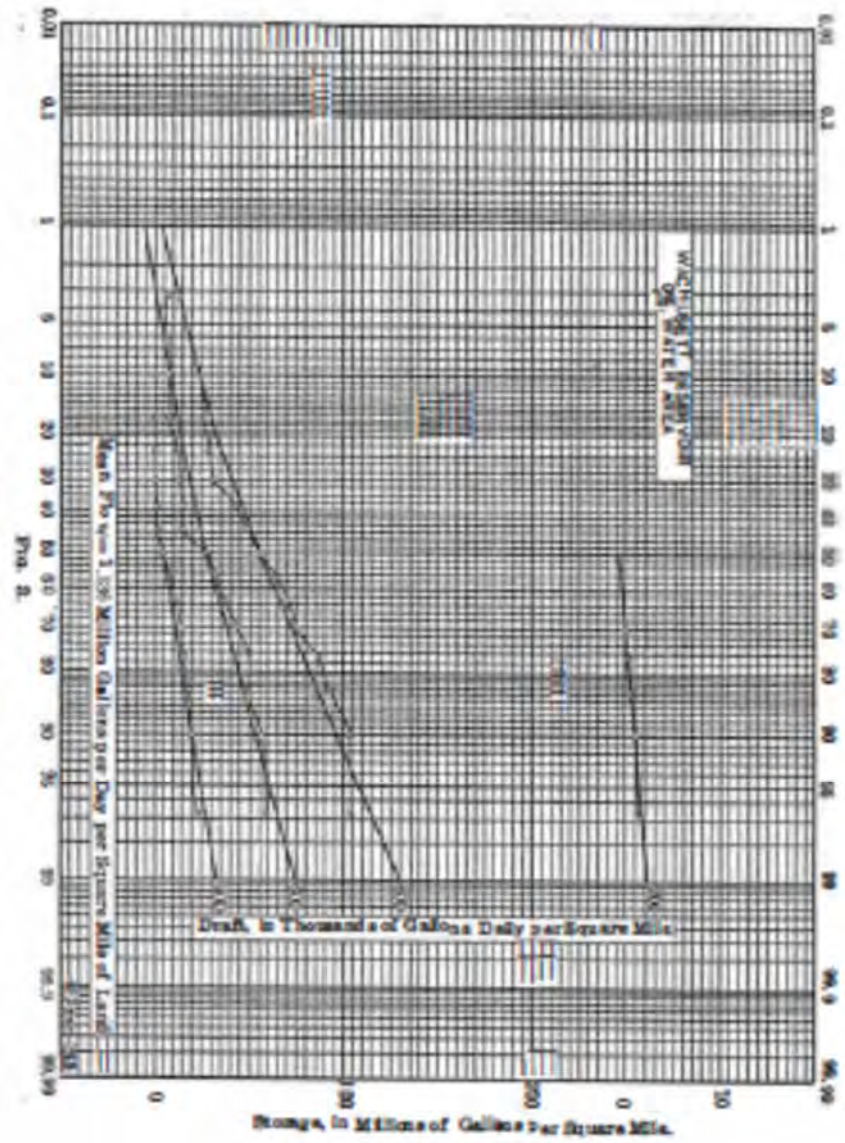
The diagrams Figs. 3 to 21, inclusive, represent all the data from the other streams mentioned, arranged in the same way; and, as these diagrams are the most convenient means of showing these data, and are sufficiently accurate, the actual figures are not presented.

In the case of three of the streams, namely, the Croton and Sudbury Rivers and the Wachusett Reservoir, three plottings are made for each. The first represents the flow as it occurred, without correction for loss by evaporation from the water area. The. second contains the storages required to balance the calculated flows for all land area. In making the correction, it has been assumed, first, that the actual flow was increased each month by the rainfall on the actual water are of the system as it existed at that time, and second, that the flow was decreased by the evaporation for that calendar month, as found by Desmond FitzGerald, Past-President, Am. Soc. C. E., in his experiments at Chestnut Hill Reservoir.* The third diagram represents the required storage to balance the calculated flows from a land area of 1 sq. mile to which there is attached 0.1 sq. mile of water surface. The allowances for rainfall and evaporation have been made in the same way.

The data have been computed and arranged in this way with the idea of showing the effect of water area on stream flow, and on the required storage; and certain deductions will be made from them as the study proceeds.

In drawing the smooth curves on the several diagrams, a uniform procedure, to be described later, has been followed, giving a definite and nearly constant curvature, this being deduced from a study of all the series here presented. The same procedure has been followed in the few cases where the data for one series alone would indicate· either more curvature or less than the line as drawn. For the streams where the records have been longest continued, the agreement of the lines with the actual records is satisfactory. For the shorter records the results are less regular, owing to the fact that they are not numerous enough to have filled in all the values that would be expected in a long-term series.

Transactiom, Am. Soc. C. E., Vol. XV, p. 581.



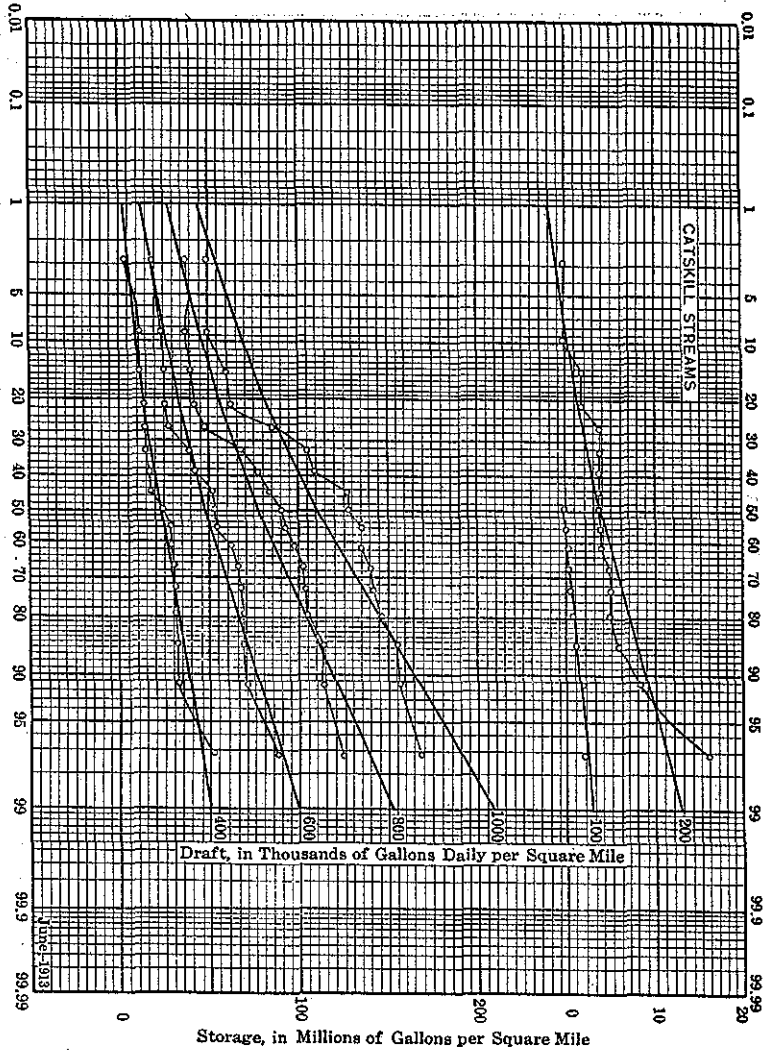

CATSKILL STREAMS
Draft, in Thousands of Gallons Daily per Square Mile
100
600
800
1000
200
100
Storage, in Millions of Gallons per Square Mile
0
100
200
0
10
20
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99
June-1913

Fig. 6.

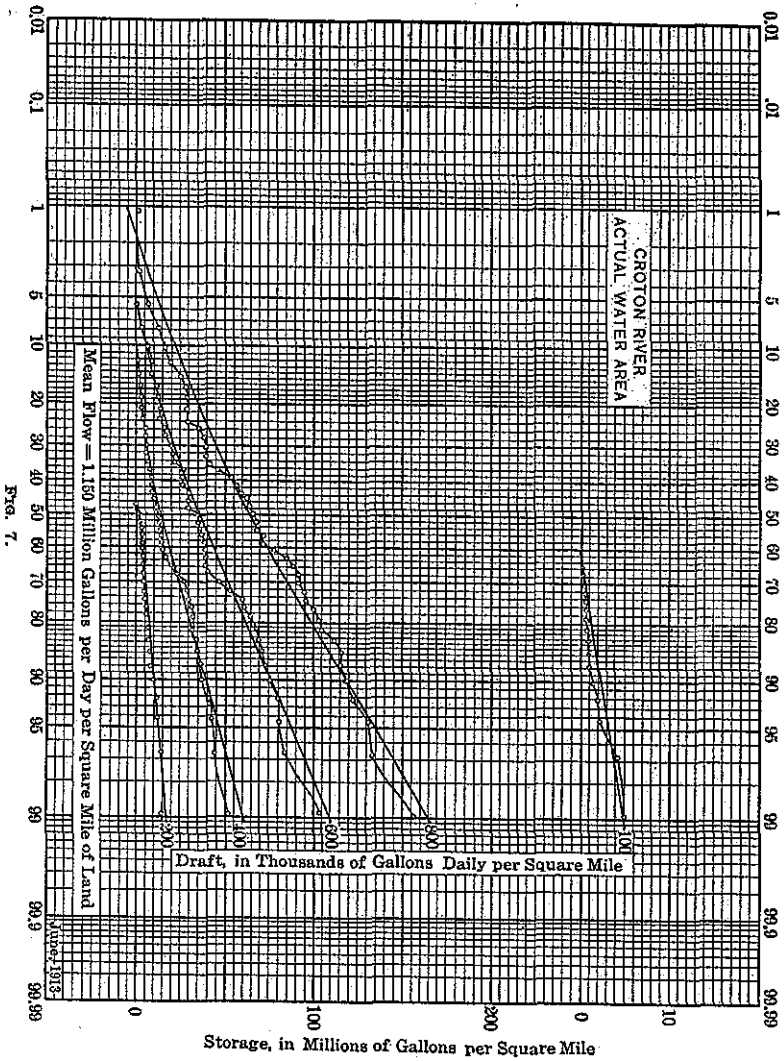

CROTON RIVER
ACTUAL WATER AREA
Mean Flow = 1.150 Million Gallons per Day per Square Mile of Land
Draft, in Thousands of Gallons Daily per Square Mile
100
600
800
1000
200
100
Storage, in Millions of Gallons per Square Mile
0
100
200
0
10
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99
June-1913

Fig. 7.

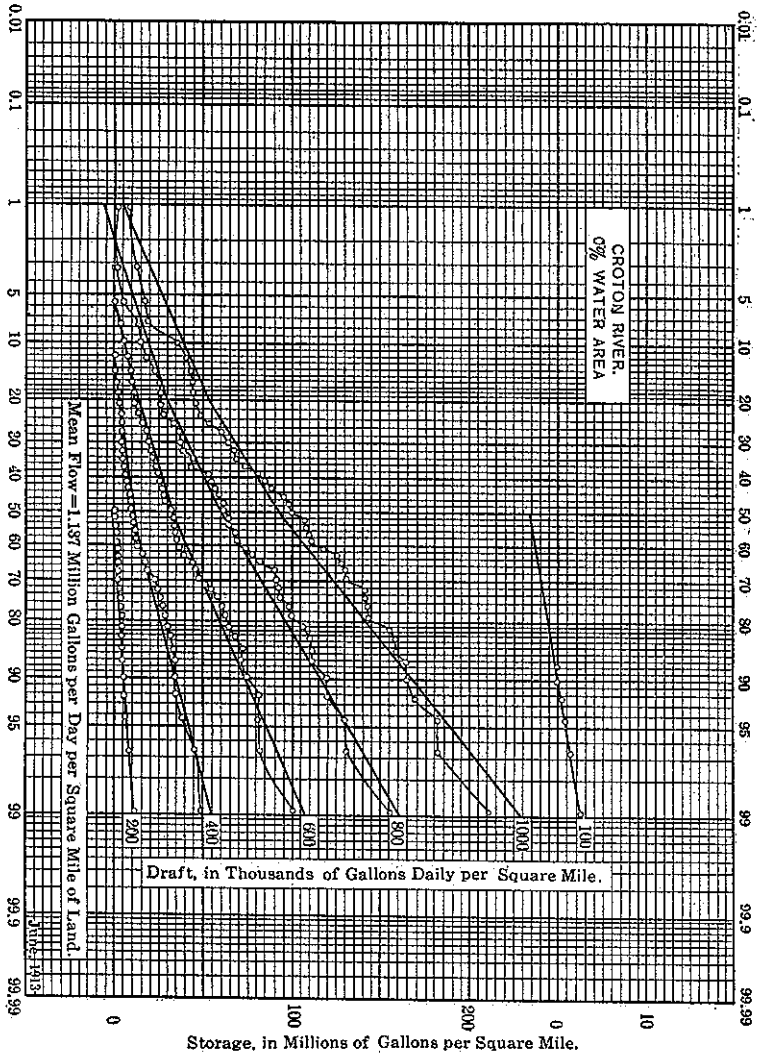

CROTON RIVER.
0% WATER AREA
Mean Flow=1.387 Million Gallons per Day per Square Mile of Land.
June 1913
Draft, in Thousands of Gallons Daily per Square Mile.
100
200
400
600
800
1000
Storage, in Millions of Gallons per Square Mile.
0
100
200
0
10
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99

Fig. 8.

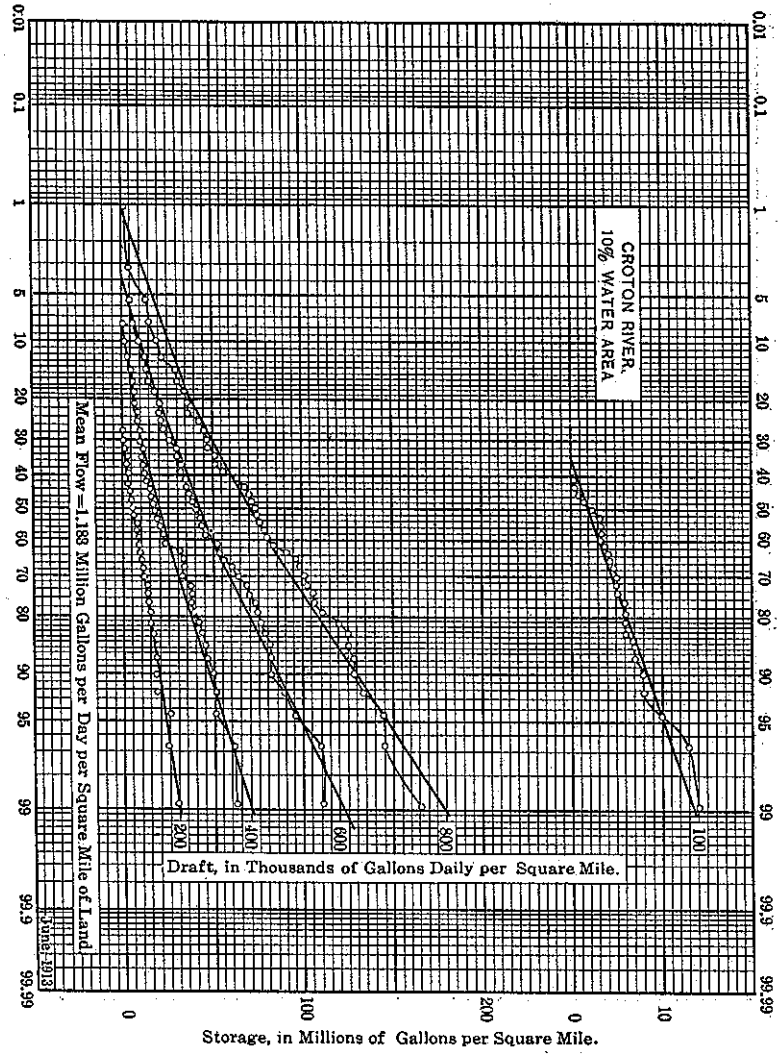

CROTON RIVER.
10% WATER AREA
Mean Flow=1.183 Million Gallons per Day per Square Mile of Land.
June 1913
Draft, in Thousands of Gallons Daily per Square Mile.
100
200
400
600
800
Storage, in Millions of Gallons per Square Mile.
0
100
200
0
10
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99

Fig. 9.

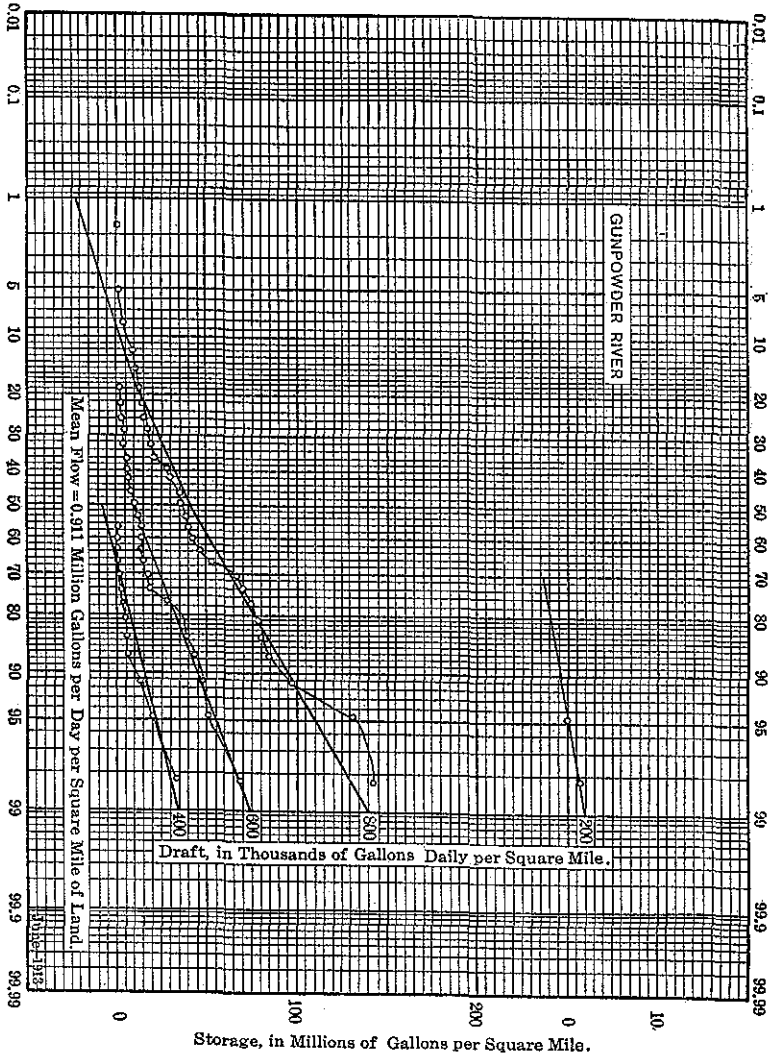

GUNPOWDER RIVER
Mean Flow = 0.911 Million Gallons per Day per Square Mile of Land.
Draft, in Thousands of Gallons Daily per Square Mile.
200
300
600
400
Storage, in Millions of Gallons per Square Mile.
0.01 0.1 1 5 10 20 30 40 50 60 70 80 90 95 99 99.9 99.99
0 100 200 0 10

Fig. 10.

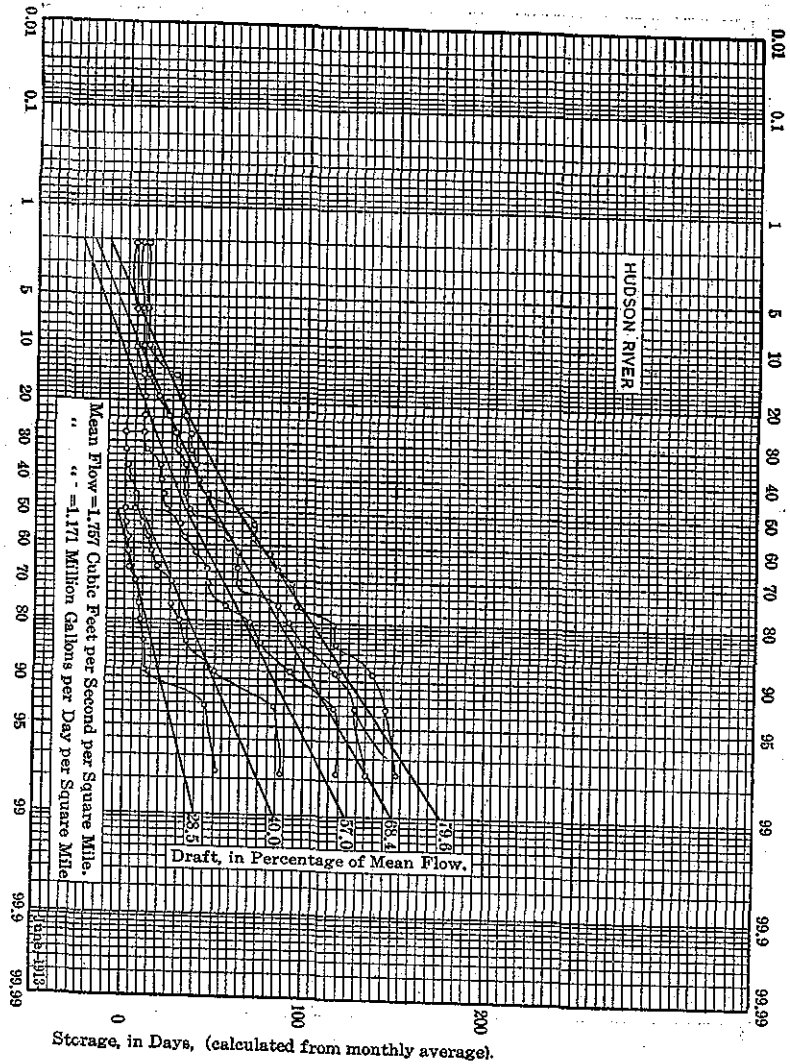

HUDSON RIVER
Mean Flow = 1.757 Cubic Feet per Second per Square Mile.
" " = 1.171 Million Gallons per Day per Square Mile
Draft, in Percentage of Mean Flow.
28.5
40.0
57.0
68.4
79.9
Storage, in Days, (calculated from monthly average).
0.01 0.1 1 5 10 20 30 40 50 60 70 80 90 95 99 99.9 99.99
0 100 200

Fig. 11.

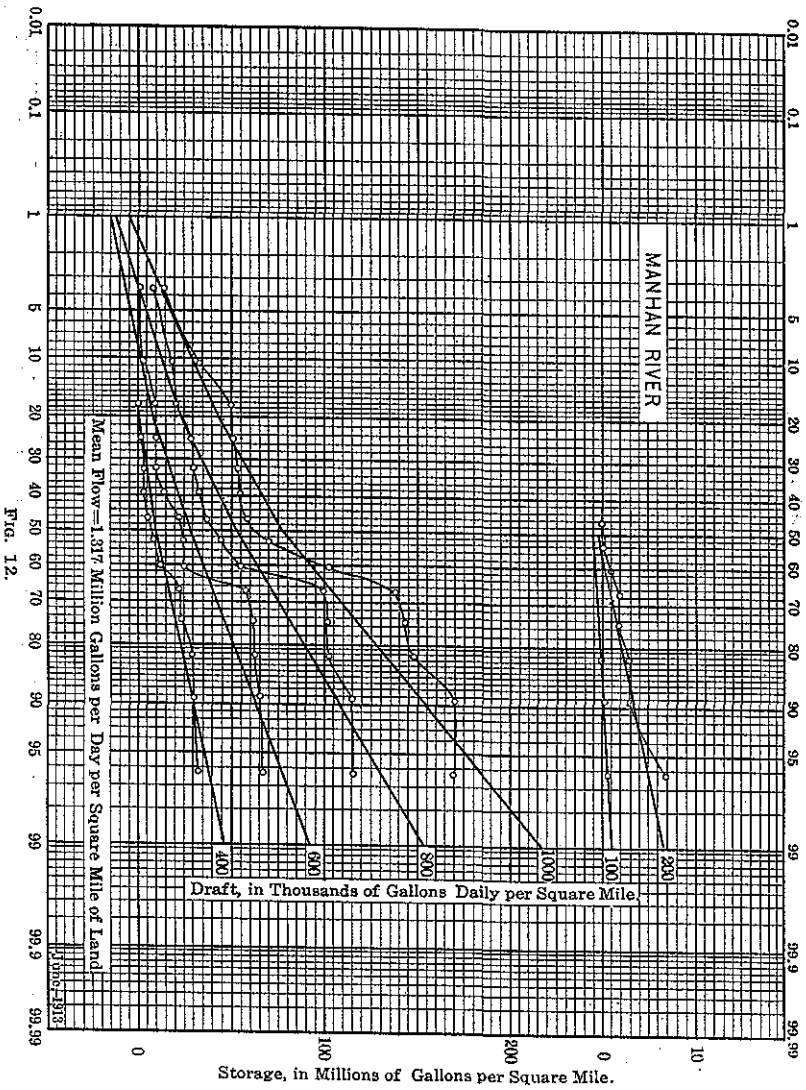

MANHAN RIVER
Mean Flow=1.317 Million Gallons per Day per Square Mile of Land
Draft, in Thousands of Gallons Daily per Square Mile.
400
600
800
1000
100
200
Storage, in Millions of Gallons per Square Mile.
0
100
200
0
10
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99
Oct-1913

Fig. 12.

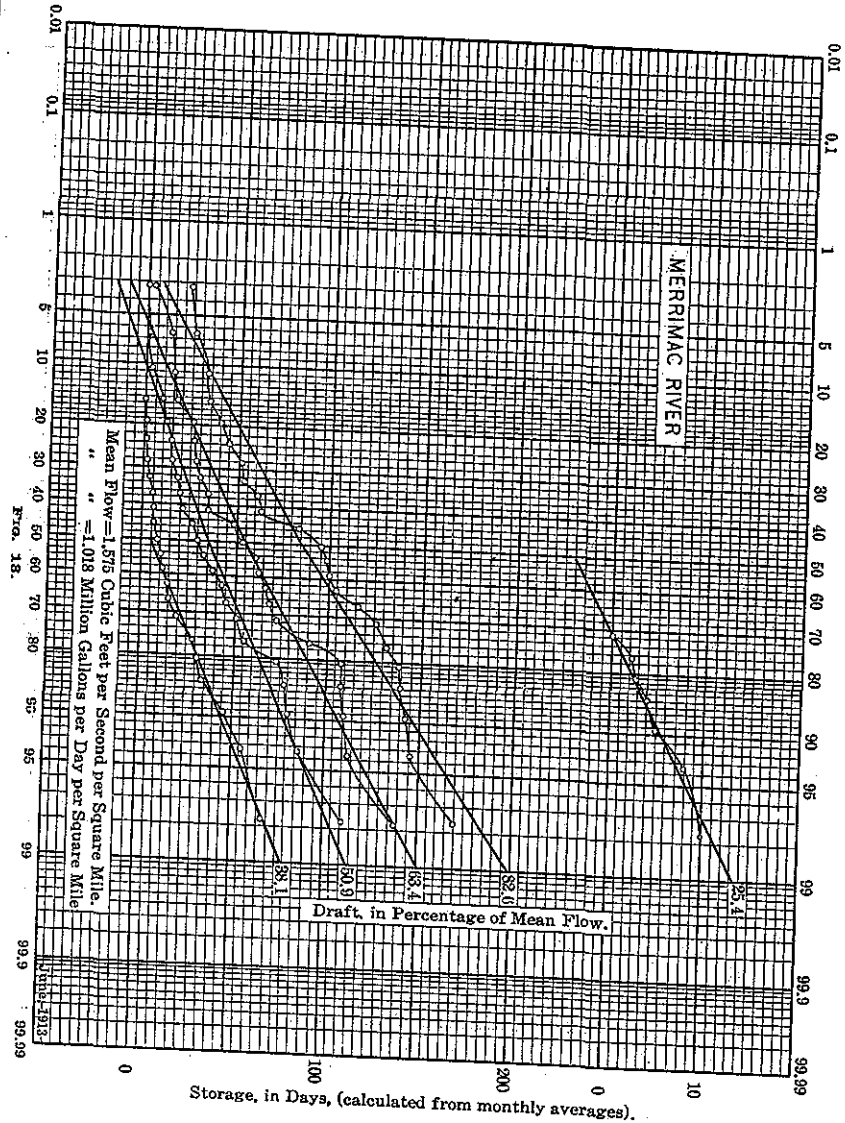

MERRIMAC RIVER
Mean Flow=1.575 Cubic Feet per Second per Square Mile.
" " =1.018 Million Gallons per Day per Square Mile
Draft, in Percentage of Mean Flow.
95.4
82.0
63.4
50.9
38.1
Storage, in Days, (calculated from monthly averages).
0
100
200
0
10
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99
Oct-1913

Fig. 13.

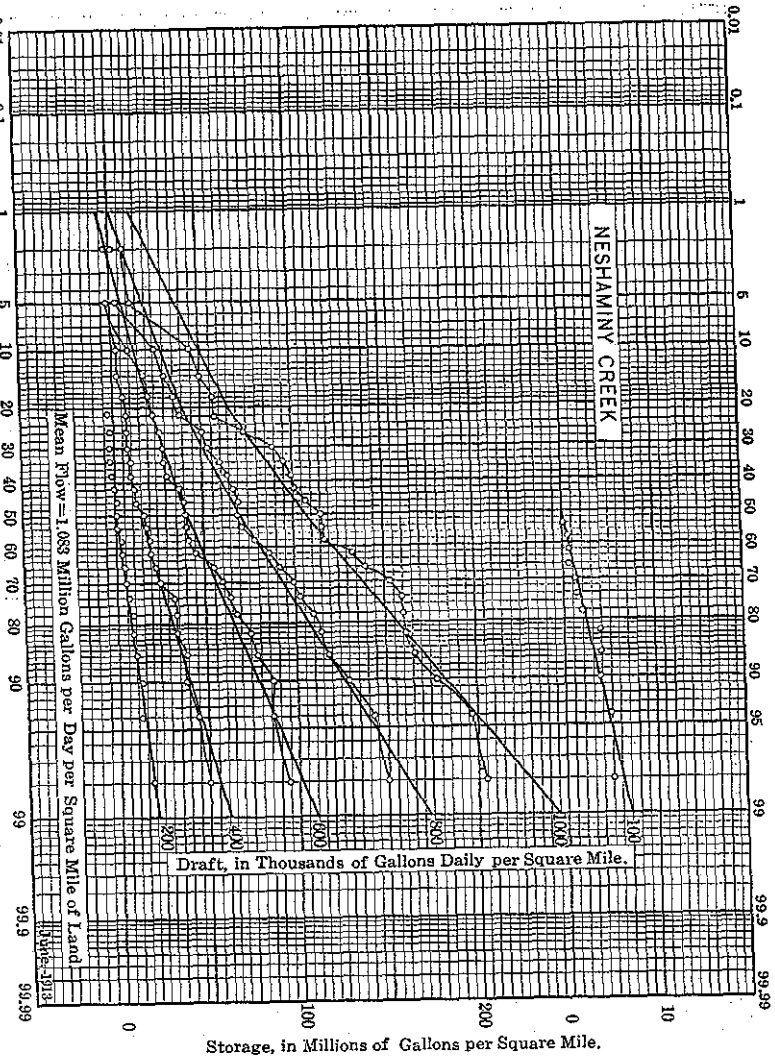

NESHAMINY CREEK
Mean Flow=1.083 Million Gallons per Day per Square Mile of Land.
Draft, in Thousands of Gallons Daily per Square Mile.
100
200
300
400
500
600
700
800
Storage, in Millions of Gallons per Square Mile.
0
100
200
0
10
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99

Fig. 14.

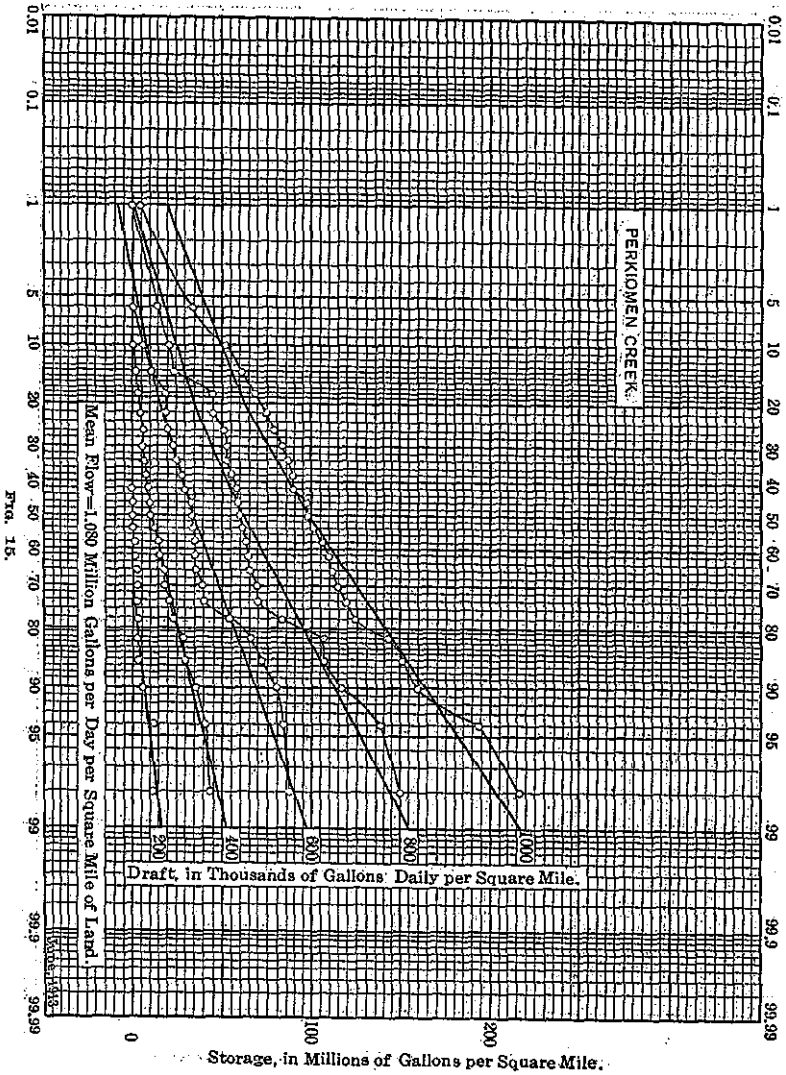

PERKIOMEN CREEK.
Mean Flow=1.080 Million Gallons per Day per Square Mile of Land.
Draft, in Thousands of Gallons Daily per Square Mile.
200
400
600
800
1000
Storage, in Millions of Gallons per Square Mile.
0
100
200
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99

Fig. 15.

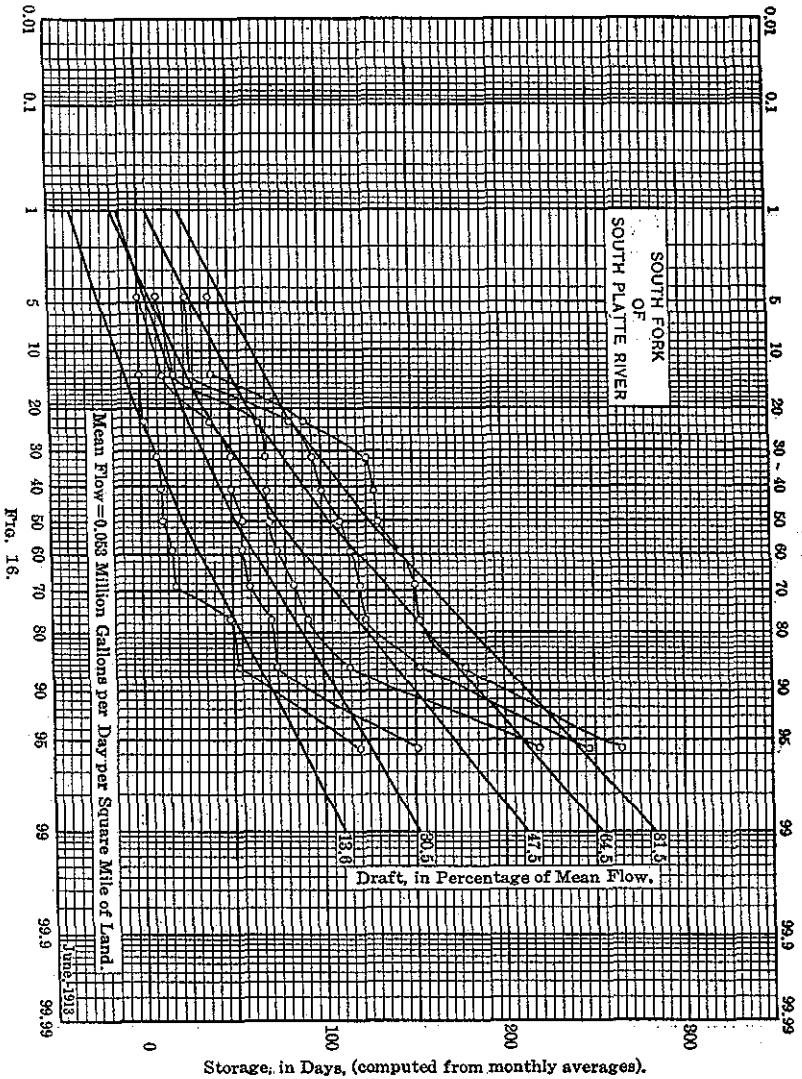

SOUTH FORK
OF
SOUTH PLATTE RIVER
Mean Flow=0.053 Million Gallons per Day per Square Mile of Land.
June 1913
81.3
64.3
47.3
20.5
13.6
Draft, in Percentage of Mean Flow.
Storage, in Days, (computed from monthly averages).

FIG. 16.

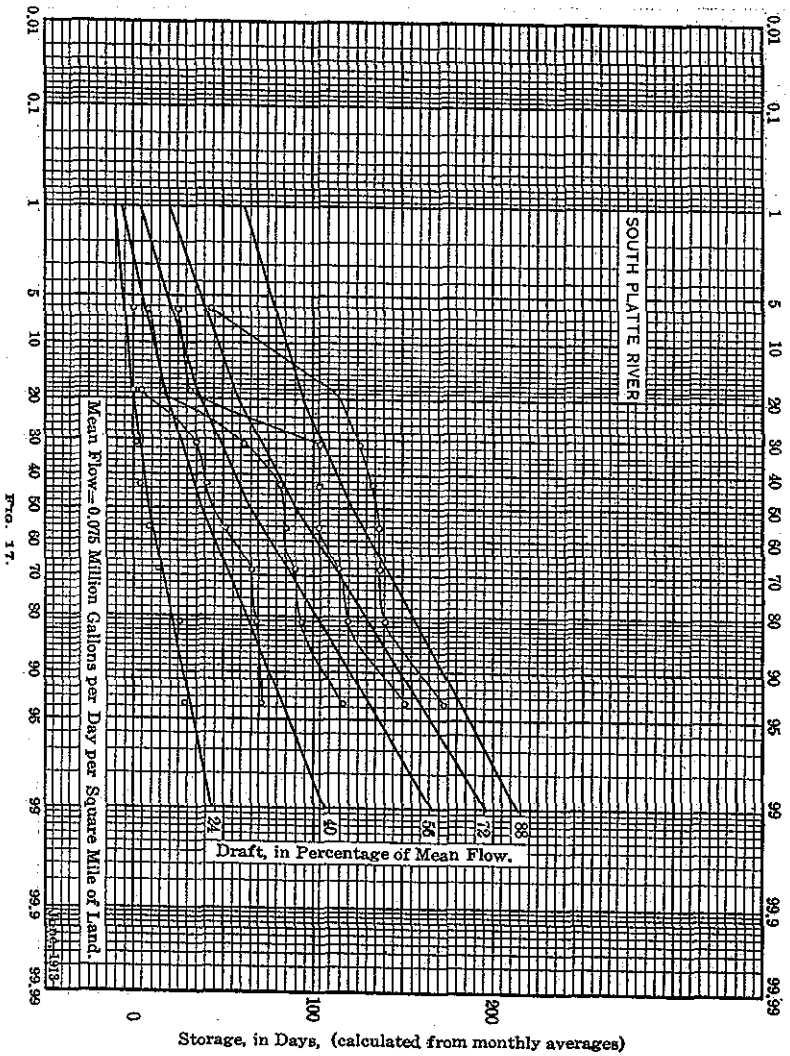

SOUTH PLATTE RIVER
Mean Flow=0.075 Million Gallons per Day per Square Mile of Land.
June 1913
88
72
56
45
24
Draft, in Percentage of Mean Flow.
Storage, in Days, (calculated from monthly averages)

FIG. 17.

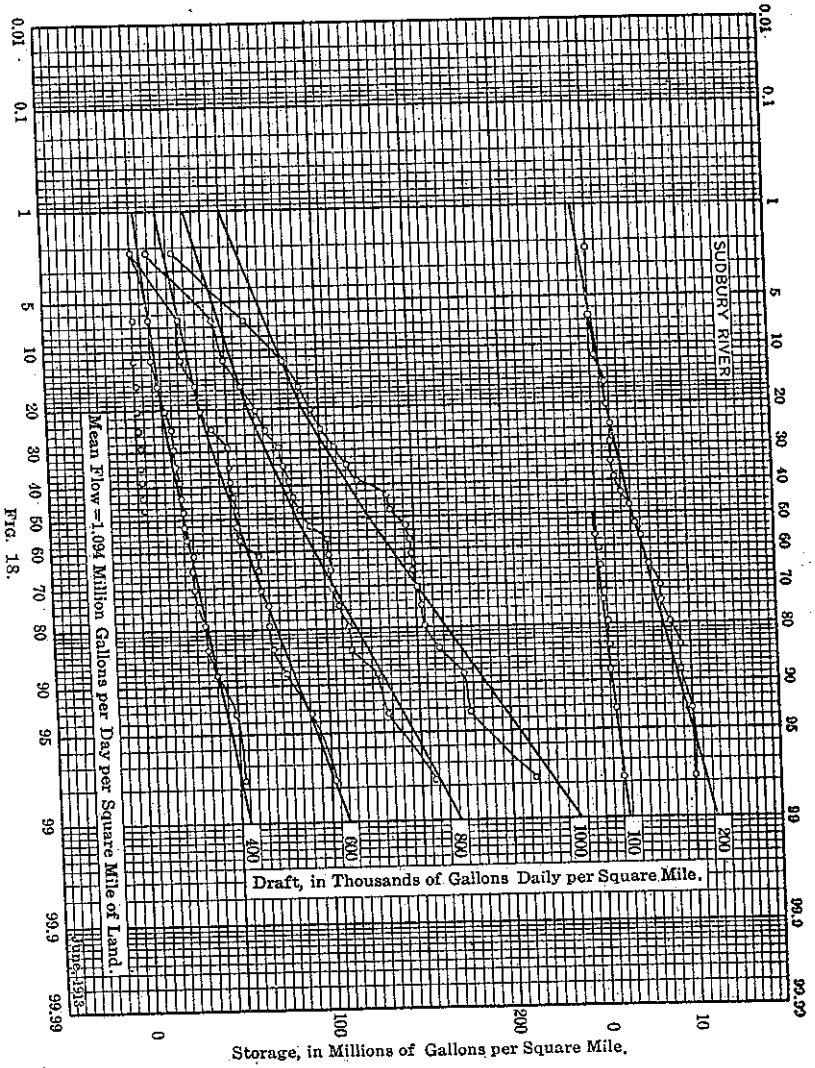

SUDBURY RIVER
Mean Flow=1.094 Million Gallons per Day per Square Mile of Land.
June, 1913.
Draft, in Thousands of Gallons Daily per Square Mile.
400
600
800
1000
100
200
Storage, in Millions of Gallons per Square Mile.
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99
0
100
200
10

FIG. 18.

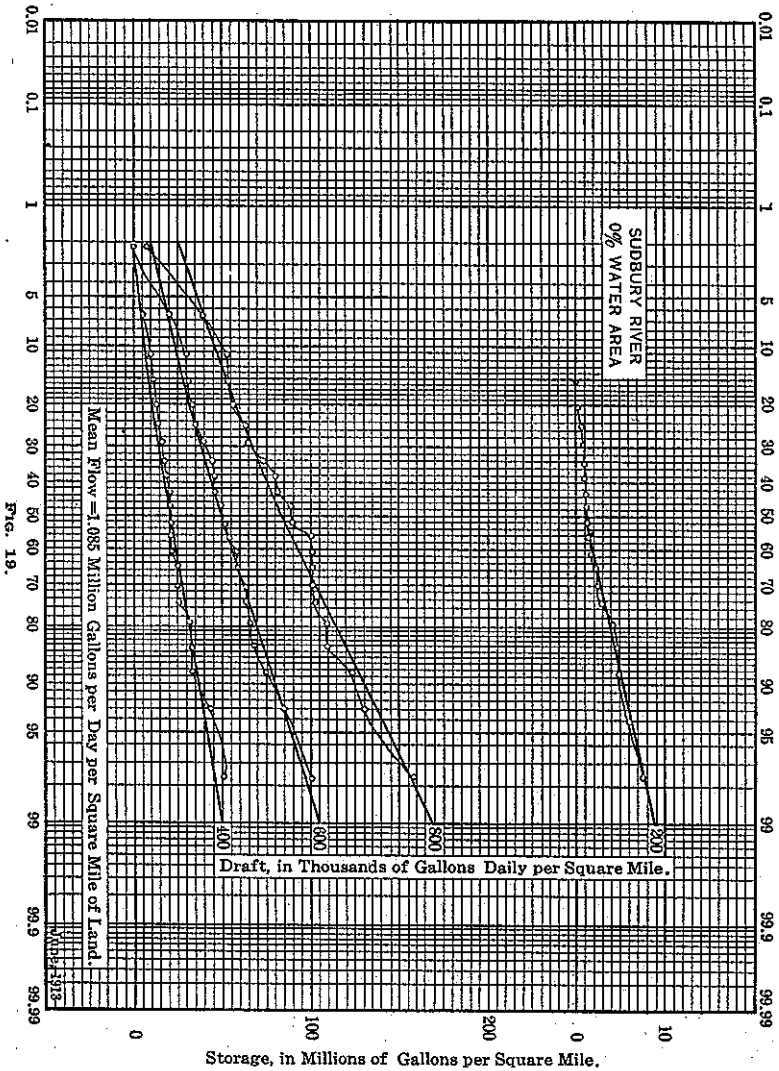

SUDBURY RIVER
0% WATER AREA
Mean Flow=1.085 Million Gallons per Day per Square Mile of Land.
June, 1913.
Draft, in Thousands of Gallons Daily per Square Mile.
400
600
800
200
Storage, in Millions of Gallons per Square Mile.
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99
0
100
200
10

FIG. 19.

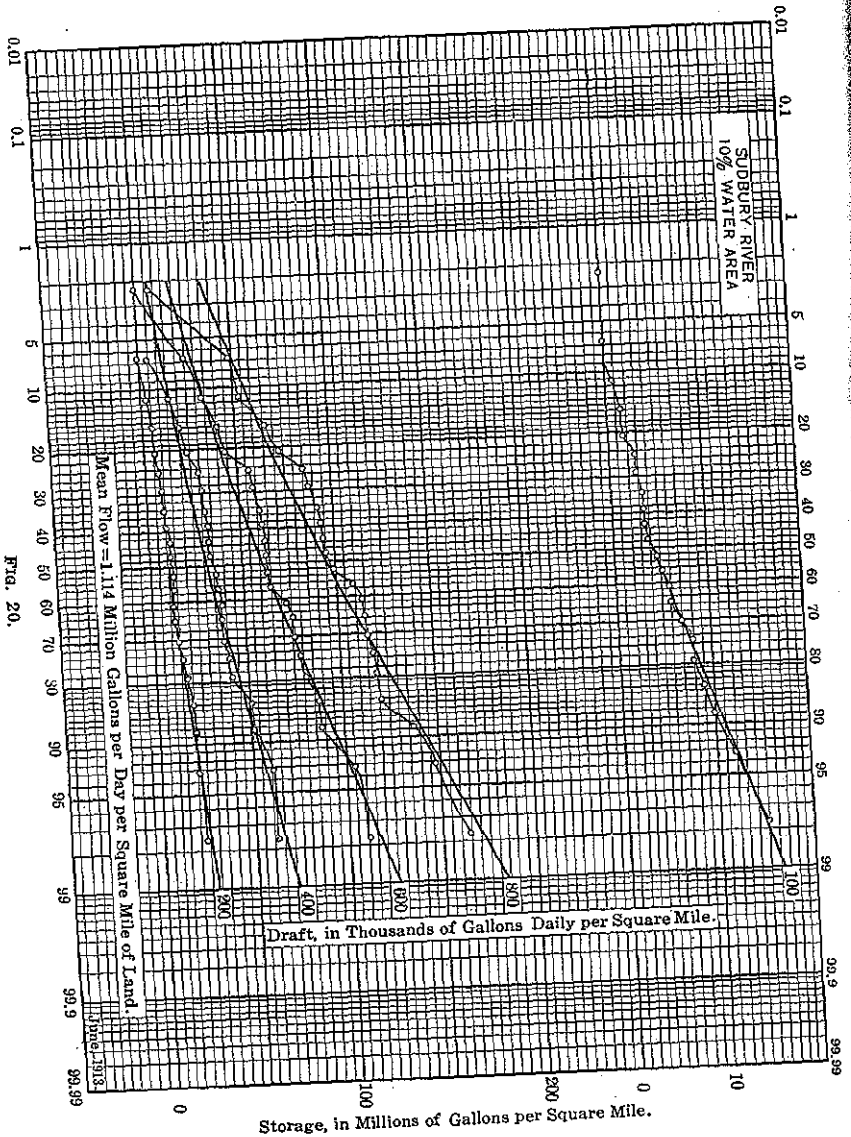

SUDBURY RIVER
10% WATER AREA
Mean Flow = 1.114 Million Gallons per Day per Square Mile of Land.
Draft, in Thousands of Gallons Daily per Square Mile.
Storage, in Millions of Gallons per Square Mile.
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99
0
10
100
200
400
600
800
1000

Fig. 20.

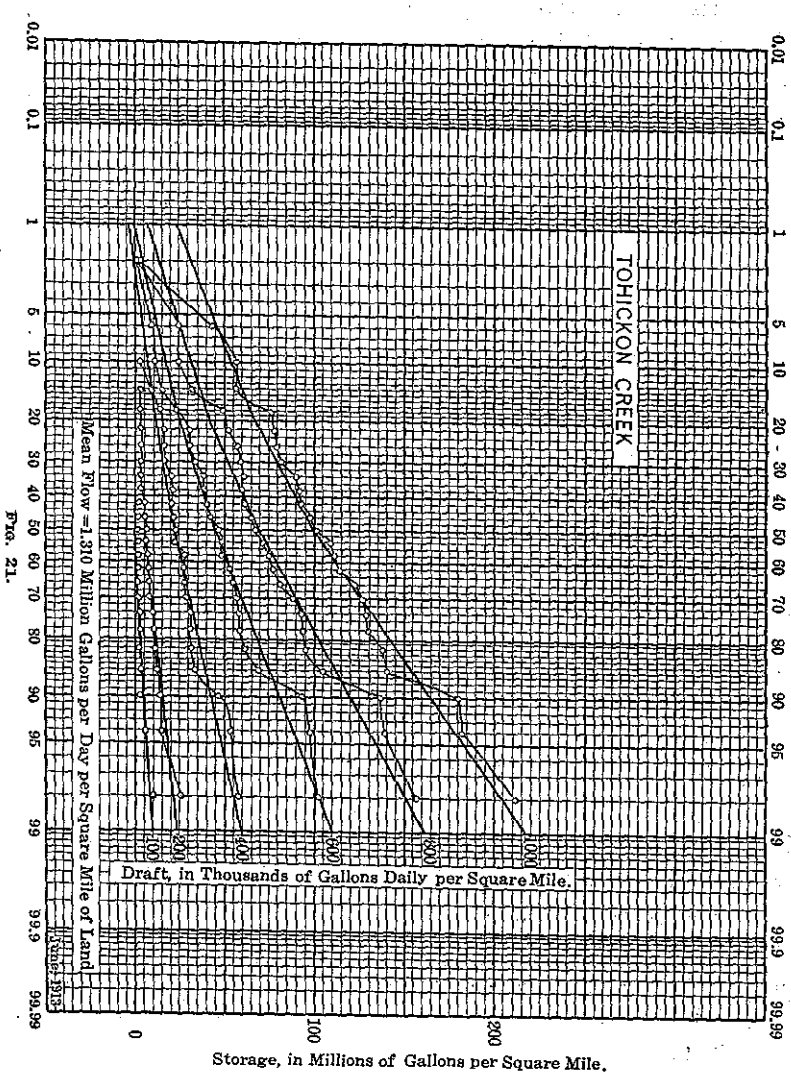

TOHICKON CREEK
Mean Flow = 1.310 Million Gallons per Day per Square Mile of Land.
Draft, in Thousands of Gallons Daily per Square Mile.
Storage, in Millions of Gallons per Square Mile.
0.01
0.1
1
5
10
20
30
40
50
60
70
80
90
95
99
99.9
99.99
0
100
200
400
600
800
1000

Fig. 21.

*Definition of a Dry Y*ear. -Dry years recur at intervals. The dryer the year the longer is the probable interval of its recurrence. For intelligent discussion, it is necessary to define a dry year in terms which will designate the degree of dryness.

In this discussion the procedure has been adopted of arranging all the years in a given series in the order of their dryness. The median year in such a series is referred to as the "50% year". The year of such a degree of dryness that 90% of the years are wetter and 10% are dryer than it, is called the "90% dry year", and the year such that 99% of all the years are wetter and 1% dryer than it, is called the "99% dry year". Years thus defined are types. No one actual year is meant.

Dry years may be classified with reference to the quantities of rainfall, the quantities of run-off, or the quantities of storage required to maintain certain drafts. Arranging all the years in series in the order of dryness on these different bases will not always place them in the same order. One year may be the dryest with reference to rainfall another with reference to run-off, and still another with reference to the maximum storage required. In this discussion such differences are overlooked, and the 90% dry year is considered as a type and always refers to the year defined as above with reference to whatever matter may be under investigation at the time.

Error by *Using Monthly* Results. -Thus far, all calculations have been made on a basis of the average monthly flows. Obviously, there will be fluctuations in flow in the days of the months at the beginning of the period of depletion, and in the month during which depletion culminates, which are not represented by the monthly averages. In order to determine how much should be added for daily variations in flow within these months, the Manhan records, for which daily records were also available, were used, and the calculations were made again on the basis of the daily flows. These figures, compared with those based on the monthly average results, are given in Table 2.

There is a slight tendency for the larger figures to occur in the drier years, so that, from a dry-year standpoint, the correction should be above the general average of 8.2 days. Nine is selected as a correction to be added to all monthly figures. This will be called the "daily storage".

Effect of Using the Monthly Basis on the Rest of .the Calculation.- It is found that the storage required, on the basis of the monthly mean figures which have been used in this and in most prior investigations, gives too little storage for periods ranging from 0 to 18 days, averaging 9 days. The quantity of excess storage for each year within these limits is a matter of chance. It depends mainly on whether the storm that terminates the drought and commences to refill the reservoir occurs early or late in the calendar month.

TABLE 2.-Additional Number of Days' Storage Required When Daily Flows Instead of Monthly Means for the Manhan River are used.

Year.	For Drafts, in Gallons per Square Mile of Land Area per Day.			
	400 000	600 000	800 000	1 000 000
1897	2	8	6	4
1898	5	3	5	6
1899	0	0	7	7
1900	5	7	3	5
1901	5	8	9	12
1902	17	8	11	13
1903	5	8	6	7
1904	10	8	12	9
1905	5	5	15	11
1906	12	12	4	5
1907	5	8	12	18
1908	10	8	10	11
1909	7	17	14	16
Average	8.8	7.7	8.8	9.5

The effect of using monthly means is to introduce an accidental variation growing out of the method of record and of calculation, in addition to all the natural variations that exist. As the average excess, called daily storage, is known approximately from the foregoing calculation, no large constant error is to be expected in the corrected result. The fact of the additional variation makes all the monthly figures so much more variable, and adds to the difficulty of analyzing them correctly.

If the matter of securing run-off data were to be taken up again, there would be much to be said in favor of weekly averages. The probably discrepancy between the required storage calculated from the

weekly averages and the daily results would be so small that it could be overlooked. The weekly averages would be easier of analysis than the daily results, and would serve all practical purposes. The weeks are also all of the same length, and the slight errors introduced by the fact that the months are not of the same length would be eliminated. As nearly all the data now available are on a monthly basis, and as the daily records could only be secured and analyzed with great labor (and, in some instances, not at all), this matter is not open for re-consideration at this time. Attention is called to it with the view of raising the question whether in future it would not be better to use the weekly instead of the monthly basis. The weekly basis has been used always for the record of the flow of the Merrimac River, kept by the Essex Company at Lawrence, for the Connecticut River at Holyoke, and the Pequannock River by the City of Newark.

The importance of making the correction for daily results, when monthly records are used for the basis of calculation, will be realized when it is stated that, for most of the Eastern streams investigated, this correction amounts to more than the allowance for evaporation.

Method of Least Squares.- Some of the methods of least squares have been found to be applicable to the data of flows and storage. Without explaining the methods found in textbooks, the following fundamental definition may be given:

The "mean" of a series of terms is the arithmetical average of all the terms. The "median" is the middle term of the series. The "variation" of any term is the difference between that term and the mean. The "standard variation" is the square root of the mean square of the variations of all the terms. The "coefficient of variation" is the ratio of the standard variation to the mean. The "average variation" is the arithmetical average of all the variations. The "probable error" is that variation which is exceeded by one-half the variations, or it is the median of the variations.

The standard variation and the coefficient of variation have generally been used as a basis for calculation. As the tables of the curve of normal error are most commonly given in terms of the probable error it is often necessary to change one to the other. With data following the law of·normal error, the ratios between the standard variation, the average variation, and the probable error are constant, and are as follows:

Standard variation. . . •.	1.000	1.253.	1.483
Average variation.....	0.798	1.000	1.183
Probable error.....	0.6745	0.8453	1.000

Data to Which This Method is Not Applicable.-This method of analysis cannot be applied safely to any series of results in which there is·more than one zero. There are many important series of this character to be investigated. Such series include the storages required to maintain low rates of draft where, in wet years, the natural flow does not fall below the assumed rate of draft, and when, therefore, no storage is required. It also cannot be applied unless all the terms of the series are available. For instance, good data may be at hand as to the flow in a certain number of dry years, but without corresponding data for intervening years of average, or more than average, flow. For both these cases other methods of handling the data must be found.

In cases of series where there are several zeros, it is possible to get a fair approximation to the standard variation by dividing the difference between the storage for the 99% year and·the·50% year. Figures thus obtained are starred in the tables which follow to distinguish them from those found by the ordinary arithmetical procedure. By this approximate method, reliable values are obtained which are needed to round out the study and would not otherwise be available.

The adjusted mean storage computed in this way, of course, is not a true mean; but it bears the relation to that part of the series which is available that the mean of the whole series would bear if the whole series were available. The reason the mean obtained by averaging the actual figures does not bear this relation is that, if the whole series were available, some of the terms would be represented by negative numbers. These negative numbers cannot be computed and are not known, but the fact that they would exist in a theoretically complete series introduces a disturbing element if the actual mean is used.

In Table 3 is presented a concise summary of the storage data taken from Figs. 2 to 21 by the foregoing methods.

* If the normal of error applied exactly, this factor would be 2.33. The value here given is deduced from actual results obtained and shown in Fig. 29.

TABLE 3.—MONTHLY STORAGE DATA FOR SEVERAL STREAMS.
(Each Year by Itself. No Cumulative Storage Included.)

(1) Stream.	(2) Assumed Rate of Draft. Gallons per day per square mile of land area.	(3) Assumed Rate of Draft. As portion of mean flow.	(4) Storage Required, in Days' Flow at Assumed Rate of Draft, with Nine Days Added to Monthly Figures to Cover Daily Fluctuations. Mean.	(5) Storage Required, in Days' Flow at Assumed Rate of Draft, with Nine Days Added to Monthly Figures to Cover Daily Fluctuations. In 95% year.	(6) Standard variation in days' storage.
Wachusett, actual	200 000	0.174	11*	46	20.2*
	400 000	0.348	40	89	27.0
	600 000	0.521	66	126	34.8
	800 000	0.695	88	157	39.0
	1 000 000	0.870	110	187	43.0
Wachusett, no water area	200 000	0.176	7*	17	5.0*
	400 000	0.352	24*	71	28.1*
	600 000	0.528	58	112	30.7
	800 000	0.704	82	145	36.4
Wachusett, 0.1 sq. mile water area	200 000	0.171	23*	88	38.5*
	400 000	0.342	53	111	32.8
	600 000	0.513	75	141	38.5
	800 000	0.684	95	165	40.8
Bear Creek	56 000	0.300	1*	32	17.7*
	75 000	0.400	31	68	21.2
	94 000	0.500	57	100	24.2
	112 000	0.600	83	127	26.1
	150 000	0.800	131	193	35.4
Catskill streams	100 000	0.071	10*	29	11.5*
	200 000	0.143	31*	60	17.3*
	400 000	0.286	69	117	27.8
	600 000	0.428	94	152	33.4
	800 000	0.572	112	174	36.1
	1 000 000	0.715	125	190	38.1
Croton, actual	100 000	0.087	3*	40	22.4*
	200 000	0.174	17*	69	28.9*
	400 000	0.348	44*	124	46.2*
	600 000	0.522	72*	154	47.5*
	800 000	0.695	93	178	49.0
Croton, no water area	200 000	0.176	7*	48	25.0*
	400 000	0.352	38*	115	44.2*
	600 000	0.527	71*	154	48.0*
	800 000	0.703	91	175	49.0
	1 000 000	0.870	108	200	52.0
Croton, 0.1 sq. mile water area	100 000	0.085	33*	110	25.7*
	200 000	0.169	44*	130	50.0*
	400 000	0.338	64*	150	50.0*
	600 000	0.507	82*	174	52.6*
	800 000	0.676	102	193	52.2
Gunpowder	200 000	0.220	—10*	9	11.6*
	400 000	0.439	— 5*	62	40.3*
	600 000	0.658	30*	100	41.0*
	800 000	0.877	65	147	46.5
Hudson†	324 000	0.285	7*	36	17.0*
	454 000	0.400	22*	71	28.5*
	648 000	0.570	47	106	34.4
	775 000	0.684	63	129	38.6
	905 000	0.796	77	152	43.2

* Adjusted values which have been inferred from the line drawn to represent the data.
† Includes water area.

TABLE 3.—(*Continued.*)

(1) Stream.	(2) Assumed Rate of Draft. Gallons per day per square mile of land area.	(3) Assumed Rate of Draft. As portion of mean flow.	(4) Storage Required, in Days' Flow at Assumed Rate of Draft, with Nine Days Added to Monthly Figures to Cover Daily Fluctuations. Mean.	(5) Storage Required, in Days' Flow at Assumed Rate of Draft, with Nine Days Added to Monthly Figures to Cover Daily Fluctuations. In 95% year.	(6) Standard variation in days' storage.
Manhan	200 000	0.152	10*	33	13.1*
	400 000	0.304	38*	99	35.6*
	600 000	0.455	61	132	41.5
	800 000	0.607	82	165	48.1
	1 000 000	0.760	99	186	51.8
Merrimac†	259 000	0.254	6*	17	6.7*
	388 000	0.381	16*	66	29.3*
	519 000	0.509	47	100	30.2
	646 000	0.634	67	132	37.2
	842 000	0.826	97	175	44.3
Neshaminy	100 000	0.092	12*	59	27.0*
	200 000	0.184	32*	90	34.6*
	400 000	0.369	56*	130	43.1
	600 000	0.554	84	160	44.0
	800 000	0.738	104	190	48.8
	1 000 000	0.924	122	214	53.0
Perkiomen	200 000	0.185	—2*	59	36.5*
	400 000	0.370	43*	109	38.5*
	600 000	0.555	70	141	40.2
	800 000	0.740	91	169	44.1
	1 000 000	0.926	112	192	45.5
South Fork of South Platte	7 200	0.136	36*	94	33.2*
	16 200	0.305	66	133	37.2
	25 000	0.475	90	181	52.5
	34 000	0.645	119	219	57.4
	43 000	0.815	141	246	60.3
South Platte	18 000	0.240	18*	43	13.8*
	30 000	0.400	45	96	26.1
	42 000	0.560	70	146	35.7
	54 000	0.720	92	175	40.8
Sudbury, actual	100 000	0.091	9*	28	10.8*
	200 000	0.183	31*	59	16.0*
	400 000	0.366	73	125	30.7
	600 000	0.548	101	164	37.3
	800 000	0.732	122	192	40.4
	1 000 000	0.915	139	215	43.2
Sudbury, no water area	200 000	0.184	16*	42	15.0*
	400 000	0.368	63	114	28.8
	600 000	0.553	94	157	36.0
	800 000	0.736	117	188	40.8
Sudbury, 0.1 sq. mile water area	100 000	0.090	56*	139	46.4*
	200 000	0.180	67*	124	32.7*
	400 000	0.359	95	159	37.3
	600 000	0.539	116	187	41.4
	800 000	0.718	132	205	42.7
Tohickon	100 000	0.076	13*	79	38.5*
	200 000	0.152	42*	99	32.7*
	400 000	0.305	69	130	35.6
	600 000	0.458	87	160	42.3
	800 000	0.610	100	178	44.4
	1 000 000	0.764	115	193	44.7

* Adjusted values which have been inferred from the line drawn to represent the data.
† Includes water area.

The figures in Table 3 have been plotted, forming three diagrams. The assumed rate of draft, as a fraction of the mean flow, as shown in Column 3, is used as a base for each. In Fig. 22 are plotted the figures in Column 4, showing the mean storage required in days' flow for each of the streams. In Fig. 23 the figures in Column 5 are plotted, showing the storage required in a 95% dry year, and Fig. 24 shows the figures in Column 6 for standard variation in days' storage. In plotting these results, the corrected figures for no water surface for the Sudbury, Croton, and Wachusett are used, as being, on the whole, most suitable for this purpose.

It is seen by inspection of Figs. 22 and 23 that the number of days' storage required in different streams varies considerably, but the lines representing the required storage at different rates of draft are rather strikingly parallel with each other. In other words, an increased rate of draft requires nearly the same increase in the quantity of storage on all the streams. This is shown further in Tables 4, 5, and 6.

TABLE 4.—DIFFERENCE BETWEEN THE AVERAGE NUMBER OF DAYS' STORAGE REQUIRED AT SEVERAL RATES OF DRAFT, AND THE STORAGE REQUIRED FOR 50% OF THE MEAN FLOW FOR DIFFERENT STREAMS FOR THE MEAN YEAR.

Stream.	Number of years in record.	Difference Between Required Storage and Storage at 50 Per Cent.					
		0.20 use.	0.30 use.	0.40 use.	0.60 use.	0.70 use.	0.80 use.
Bear Creek	11	....	56	26	26	50	74
Catskill	17	57	31	14	11	21	30
Croton	45	55	38	19	13	25	36
Gunpowder	29	....	....	....	16	32	48
Hudson	23	....	27	14	15	29	41
Manhan	14	48	30	14	14	25	36
Merrimac	23	45	35	25	16	32	48
Neshaminy	25	42	28	15	13	24	34
Perkiomen	25	60	36	15	13	25	36
South Fork	11	46	29	15	17	32	45
South Platte	8	29	32	15	15	29	46
Sudbury	22	66	40	18	14	26	38
Tohickon	25	41	23	11	8	18	27
Wachusett	15	44	34	20	15	28	42
Weighted average	293	51.1	33.5	17.0	14.1	27.2	39.4

Table 7 shows the number of days' storage required to maintain a 50% draft in an average year and in a 95% dry year, scaled from Figs. 22 and 23, and the additional quantity required for the latter over the former.

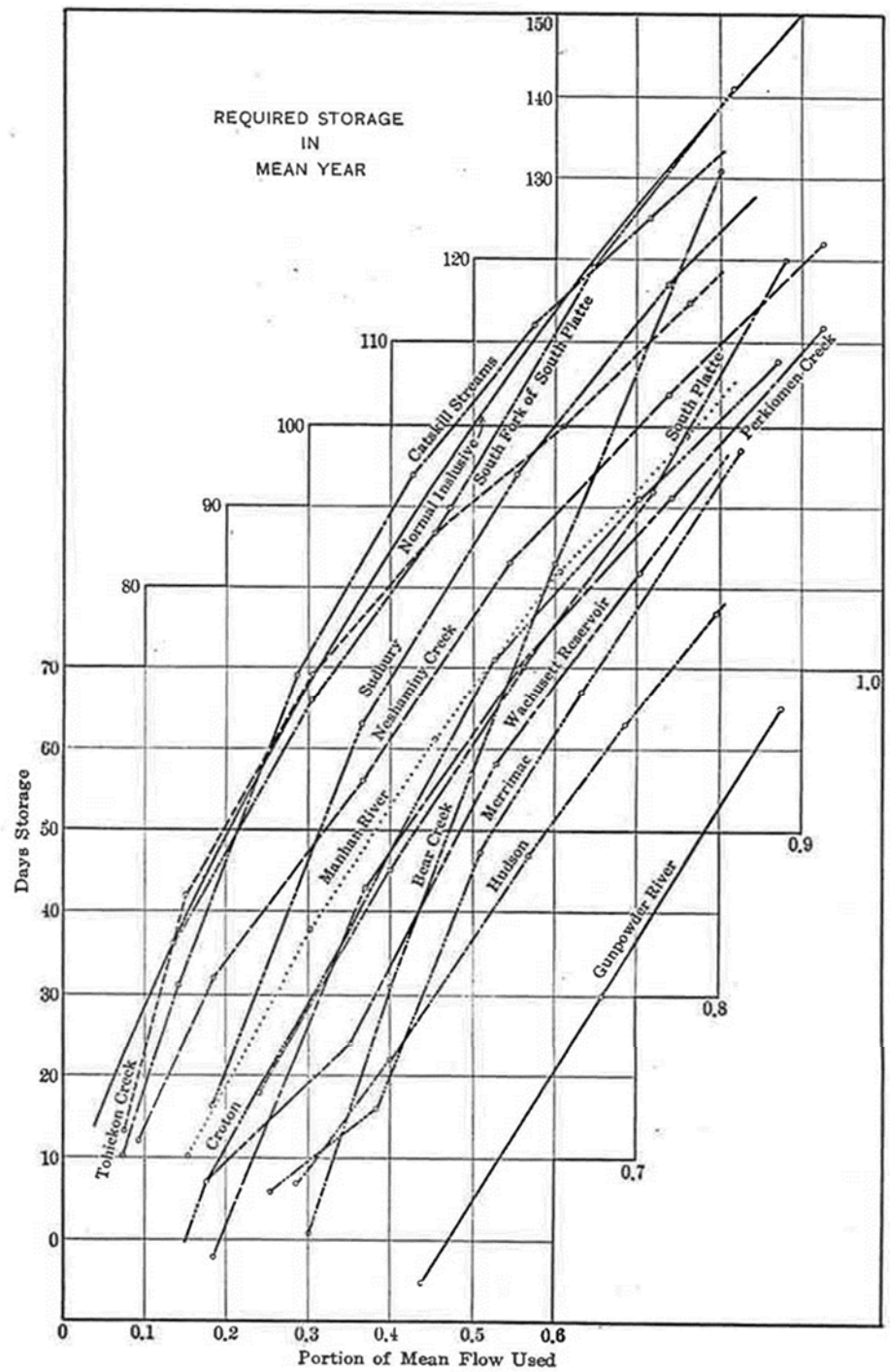


FIG. 22.

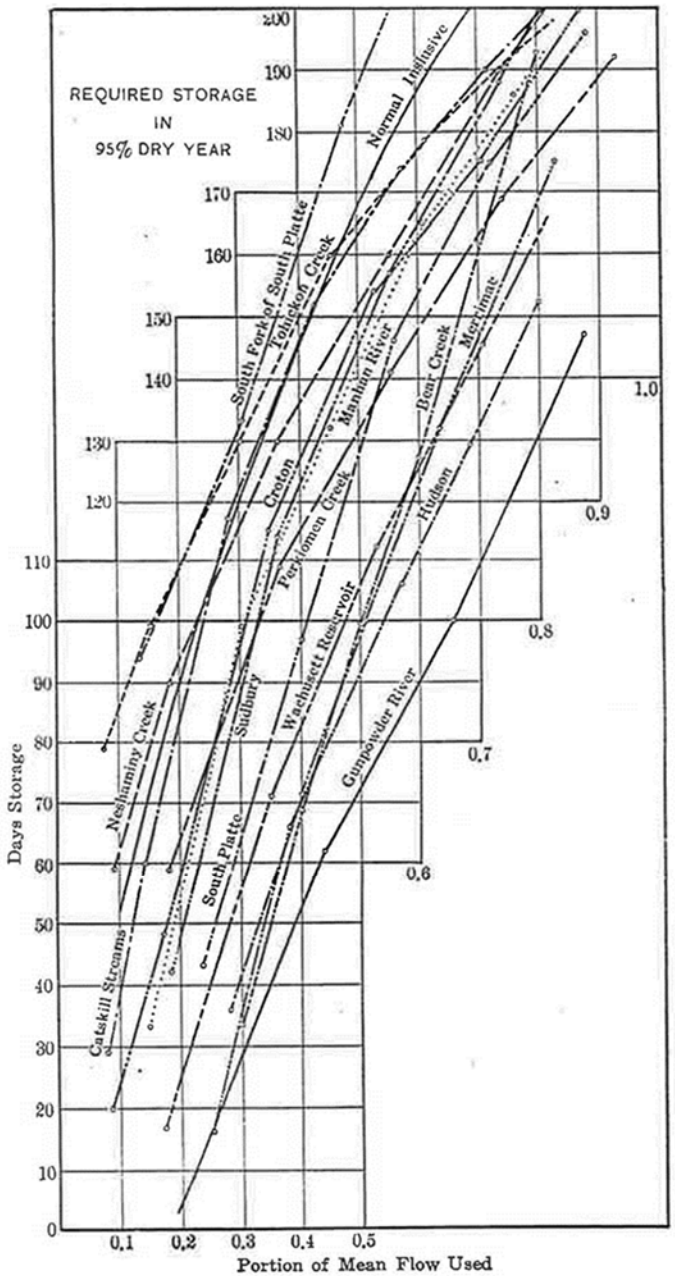

REQUIRED STORAGE
IN
95% DRY YEAR
Days Storage
Portion of Mean Flow Used
Normal Inclusive
South Fork of South Platte
Tohickon Creek
Manhan River
Bear Creek
Merrimac
Croton
Perkiomen Creek
Hudson
Neshaminy Creek
Sudbury
Washusett Reservoir
Gunpowder River
South Platte
Catskill Streams

FIG. 23.

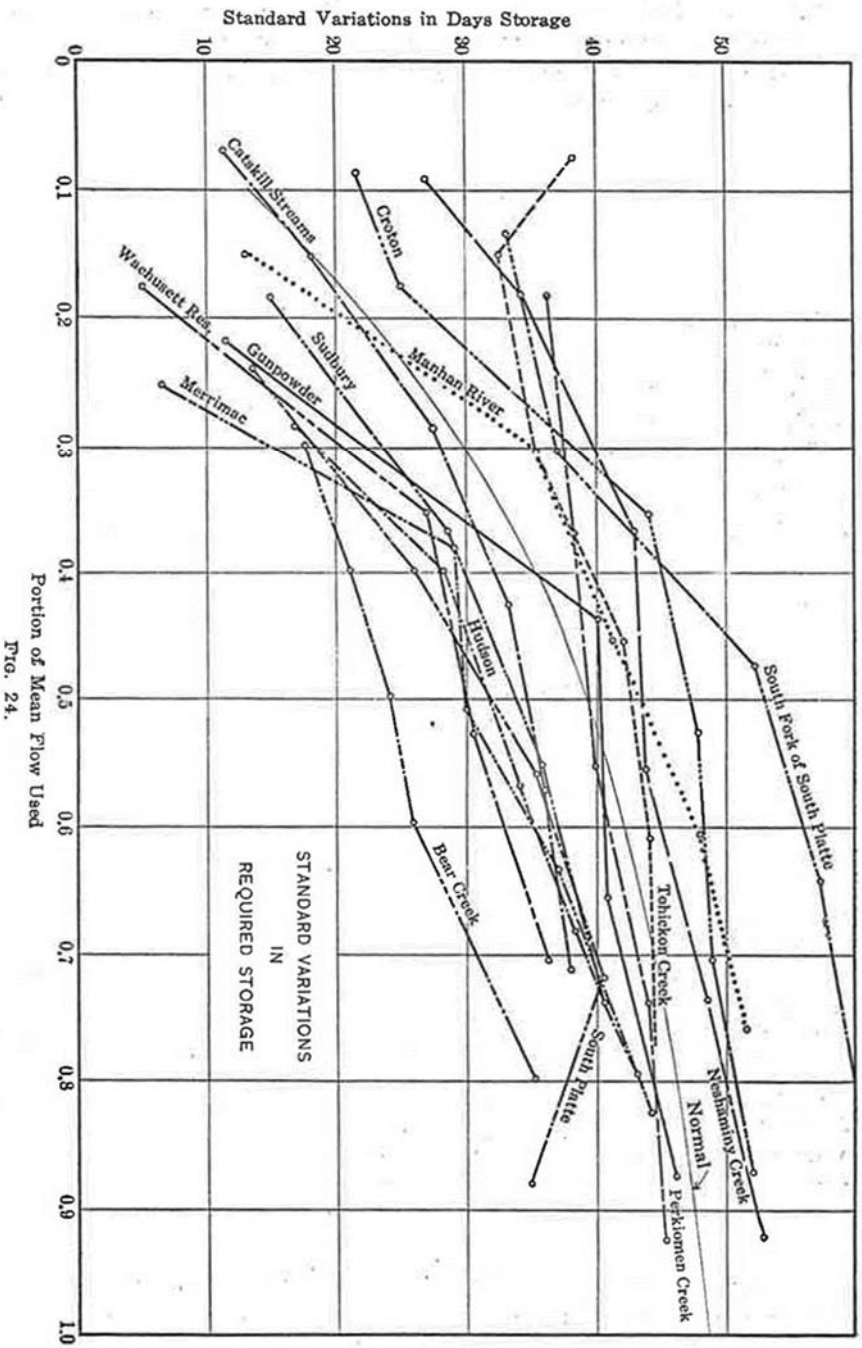

Standard Variations in Days Storage
Portion of Mean Flow Used
STANDARD VARIATIONS
IN
REQUIRED STORAGE
Catskill Streams
Croton
Washusett Res.
Sudbury
Manhan River
Gunpowder
Merrimac
Hudson
Bear Creek
South Fork of South Platte
Tohickon Creek
Neshaminy Creek
Normal
Perkiomen Creek
South Platte

FIG. 24.

TABLE 5-Difference Between the Average Number of Day's Storage Required at Several Rates of Draft, and the Storage Required for 50% of the Mean Flow for Several Streams, for the 95% Dry Year

Stream.	Number of years in record.	Difference Between Required Storage and Storage at 50 Per Cent.					
		0.20 use.	0.30 use.	0.40 use.	0.60 use.	0.70 use.	0.80 use.
Bear Creek	11	100	68	32	27	60	98
Catskill	17	81	43	18	14	25	36
Croton	45	92	54	23	14	26	38
Gunpowder	29	68	44	20	17	36	58
Hudson	23	81	51	20	21	41	62
Manhan	14	89	45	22	21	36	50
Merrimac	23	98	64	28	25	49	71
Neshaminy	25	58	36	16	17	33	47
Perkiomen	25	70	42	18	16	31	44
South Fork	11	77	55	27	23	42	57
South Platte	8	98	65	31	26	44	58
Sudbury	22	96	57	24	20	38	54
Tohickon	25	57	36	17	11	21	31
Wachusett	15	82	51	24	19	38	57
Weighted average	293	80.6	49.6	22.0	18.2	35.2	51.7

TABLE 6.—Standard Variations in Storage in Terms of Days' Flow for Several Streams at Various Rates of Draft.

Stream.	Number of years in record.	Standard Variation in Days' Flow.						
		0.20 use.	0.30 use.	0.40 use.	0.50 use.	0.60 use.	0.70 use.	0.80 use.
Bear Creek	11	....	18	21	24	26	31	35
Catskill	17	21	28	32	35	37	38	39
Croton	45	28	38	45	47	48	49	50
Gunpowder	29	9	22	36	40	41	42	45
Hudson	23	....	18	28	32	35	39	43
Manhan	14	20	35	39	43	48	50	53
Merrimac	23	....	15	29	30	35	40	43
Neshaminy	25	35	40	43	44	46	48	50
Perkiomen	25	37	38	39	40	41	43	44
South Fork	11	35	37	46	53	56	58	60
South Platte	8	12	19	26	32	37	40	38
Sudbury	22	16	24	30	34	37	40	42
Tohickon	25	34	35	40	42	44	45	45
Wachusett	15	8	21	28	30	33	36	39
Weighted average.	293	24.4	29.0	36.0	38.0	41.1	43.2	45.3

Normal Storage Diagram.- The data in Tables 4 to 7 may be combined to form a composite storage curve representing all the data. The variations in the several cases do not differ widely, but the

absolute quantities do, depending on natural storage and other conditions which are not the same for different streams. The composite curve may be an average or an inclusive one. The latter is selected. and 100 days' storage for a use equal to 50% of the normal flow is taken as the starting point. This covers all but one of the figures in Table 7, and is within the probable error of that one. The exact value taken is of no particular significance, as will appear as the method of use is developed. In a 95% dry year the normal storage (Table 7) will be 68 days more than this, or 168 days. Adding or subtracting the normal differences shown in Tables 4 and 5, to or from 168, gives the figures in Table 8.

TABLE 7.

Stream.	Number of years.	Number of Days' Storage Required to Maintain Flow Equal to 50% of Normal		
		Mean.	95% year.	Difference.
Bear Creek	11	57	100	43
Catskill	17	103	163	60
Croton	45	66	148	82
Gunpowder	29	5	72	67
Hudson	23	36	91	55
Manhan	14	67	142	75
Merrimac	23	45	98	53
Neshaminy	25	62	132	70
South Fork	11	94	186	92
South Platte	8	60	127	67
Sudbury	22	86	144	58
Tohickon	25	91	165	74
Wachusett	15	53	106	53
Weighted average	293	....	....	68

TABLE 8.

Portion which use is of mean flow.	Normal days' storage in average year.	Normal days' storage in 95% dry year.
0.10	(27)	(50)
0.20	49	88
0.30	67	119
0.40	84	145
0.50	100	168
0.60	114	186
0.70	127	203
0.80	139	219
0.90	(150)	(233)
1.00	(160)	(246)

The figures in Table 8 are plotted on probability paper (Fig. 25). The figures for the mean year are plotted on the 54% line, instead of the 50% line, because the curve is a skew curve, and investigation shows

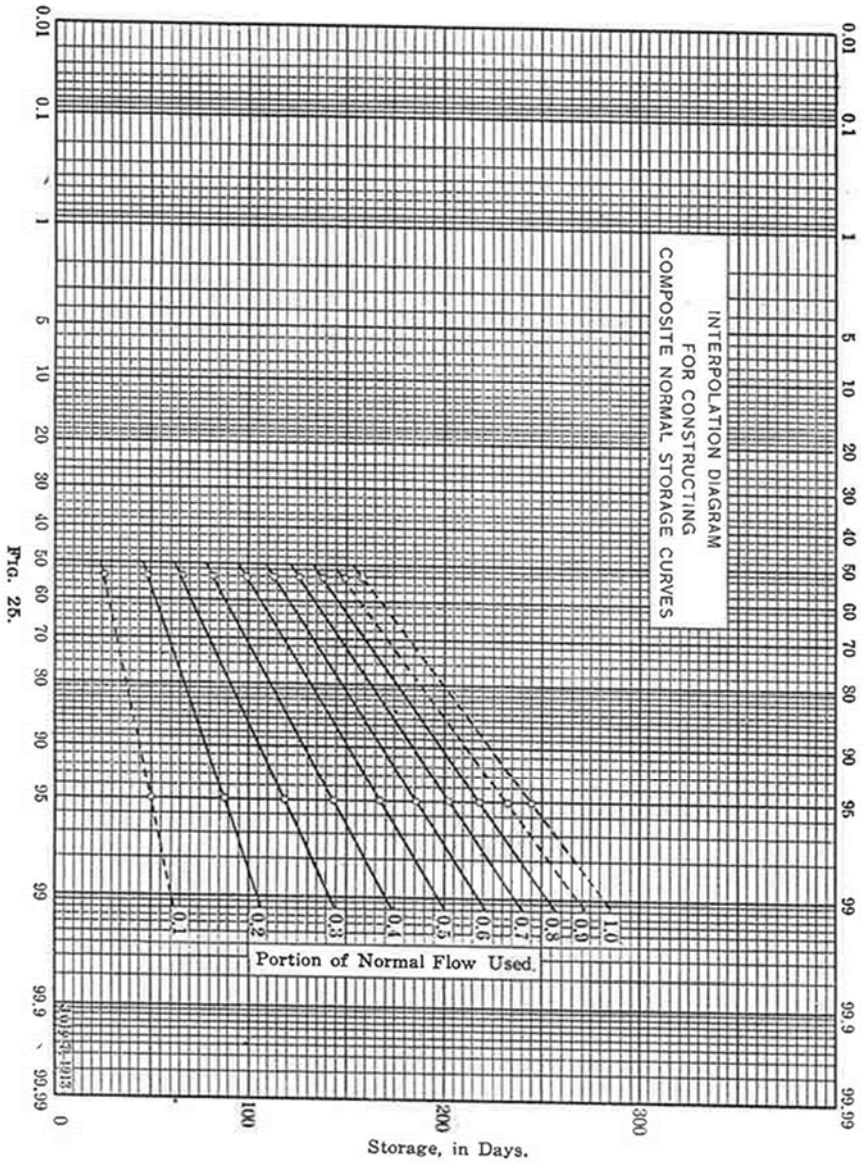


Fig. 25.

that about 54% of all the terms are less than the mean. From this diagram the number of days' storage required to maintain the several assumed rates of flow in years of other degrees of dryness may be taken. Values thus found have been used in plotting Fig, 26, which shows the normal inclusive quantity of storage required to maintain various rates of draft, in years of different degrees of dryness. This diagram, from the method of its construction, is above all but a few individual results.

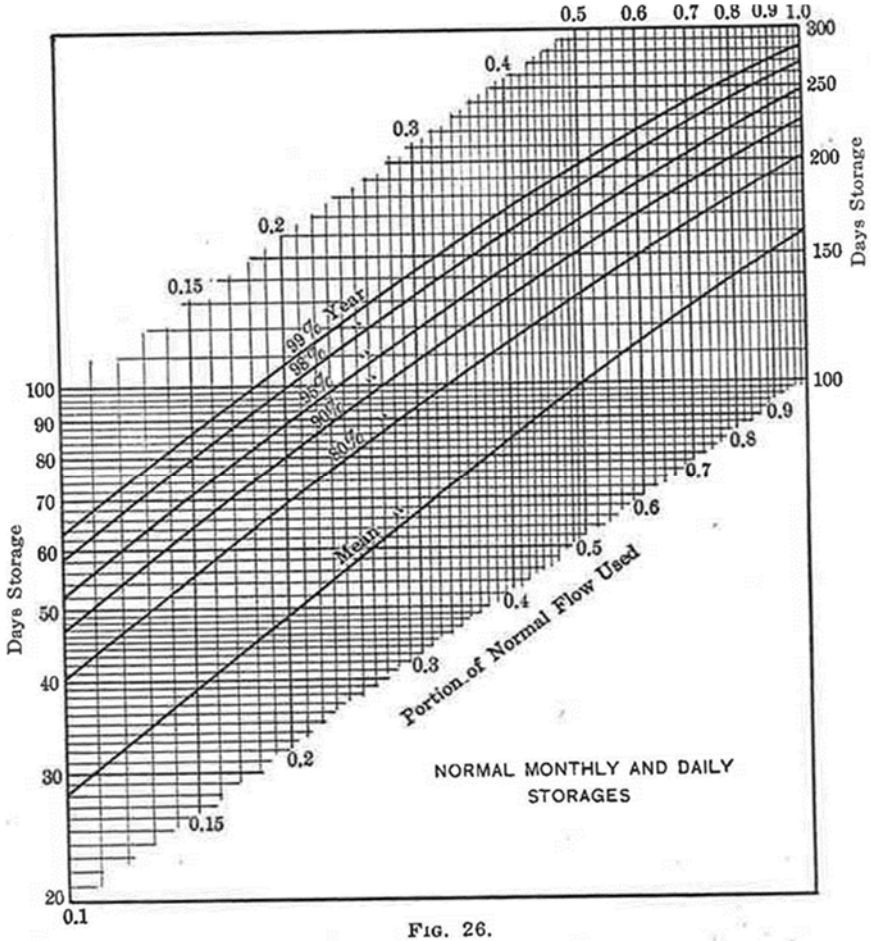
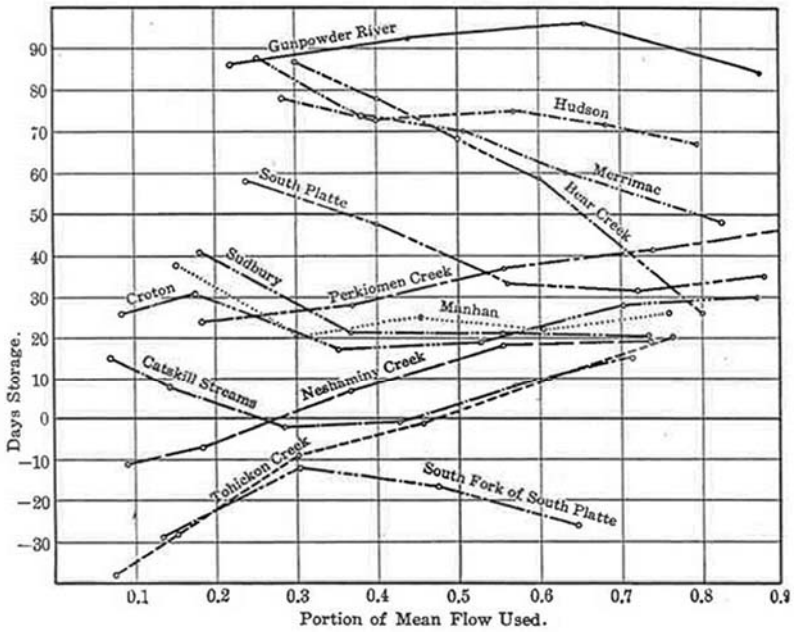


Fig. 26.

The quantity, as shown by the records of various streams, by which the storage actually required falls below the normal storage diagram at various points may be found. Table 9 shows the quantity that they fall below in a 95% dry year, and. Fig. 27 shows. the difference graphically.

If the normal storage diagram applied strictly to all the records, the figures for Table 9 for each stream at different rates of draft would be constant. This is only approximately the case. The variations represent, in part, accidental variations of the kind that would be found in different parts of a very long record of one stream, and, in part, actual difference in conditions in the different catchment areas, and in their climates and conditions of natural storage, which tend to modify the values of the normal storage diagram. The accidental



QUANTITIES WHICH ACTUAL STORAGES ARE LESS THAN NORMAL STORAGES

Showing effect of ground-water storage and other conditions peculiar to the several areas.

FIG. 27.

variations are considerable. Some of the most striking variations are in the short-term records, and these are to be considered as mainly accidental. The long-term records usually show smaller variations. However, the three Philadelphia streams, Neshaminy, Perkiomen, and Tohickon, show a well-defined tendency to require a greater relative quantity of storage for low rates of draft. This may indicate a greater summer evaporation and a smaller summer run-off, or some variation in seasonal distribution of rainfall. On the other hand, the Merrimac

requires less relative storage for low rates of draft. This may be accounted for in part by the natural lakes on this river, which modify the conditions of runoff bu the added evaporation from the water surface, and increase the flow by natural storage in the lakes, which normally runs out gradually during the summer with falling lake level, and tends to maintain the flow.

Table 9. – Quantity that the actual monthly storage (corrected for daily storage) is less than the normal storage diagram, in days' storage for several streams and rates of draft, 95% dry year.

Stream.	Number of years in record.	Difference Between Actual Required Storage and Normal Storage Diagram.							
		0.20 use.	0.30 use.	0.40 use.	0.50 use.	0.60 use.	0.70 use.	0.80 use.	Average.
Bear Creek.....	11	....	87	78	68	59	43	26	64
Catskill.....	17	4	−2	−1	4	9	15	....	5
Croton	45	29	21	18	19	23	28	....	23
Gunpowder	29	85	88	91	93	95	94	88	91
Hudson.....	23	....	78	73	75	74	71	67	73
Manhan	14	33	21	24	25	22	25	27	25
Merrimac.....	23	....	84	73	70	63	56	50	66
Neshaminy.....	25	−6	2	9	15	19	19	....	10
Perkiomen.....	25	24	26	29	34	38	40	43	33
South Fork.....	11	−22	−12	−16	−19	−24	−25	....	−20
South Platte....	8	....	55	48	39	33	32	33	40
Sudbury.....	22	40	29	22	21	21	21	20	25
Tohickon.....	25	−22	−10	−4	2	9	16	....	2
Wachusett.....	15	63	62	62	62	61	59	....	61

The area of water surface makes a substantial difference in the required storage for the lower rates of draft, but much less difference relatively for the higher rates. This is illustrated by Fig. 28, which shows the storage data for the Croton River, using the flows as they naturally occurred, as corrected to represent an area with no water, and as computed for one part of water area for each ten parts of land area, the values for this plotting being taken from Table 3.

Summary as to Monthly and Daily Storage, Excluding Annual Storage.-The foregoing studies relating to the storage required to maintain flows through one year, and excluding all consideration of cumulative storage, show that, after due allowance is made for the instantly. recurring fluctuations, depen ing on rainfall and other conditions, to analyze which no attempt is made, there remains a fairly definite and simple relation between draft and the required

storage. There is also a fairly definite and simple relation between the available supplies in years of different degrees of dryness and the frequency of the recurrence of such years. These relations may be expressed in a normal storage diagram which shows the storages in terms of days' draft. The storages thus shown are greater than those actually require on any particular stream by a number of days which is nearly constant for that stream, but varies considerably for different streams, as it depends on the natural storage on the catchment area, and other physical conditions of that stream.

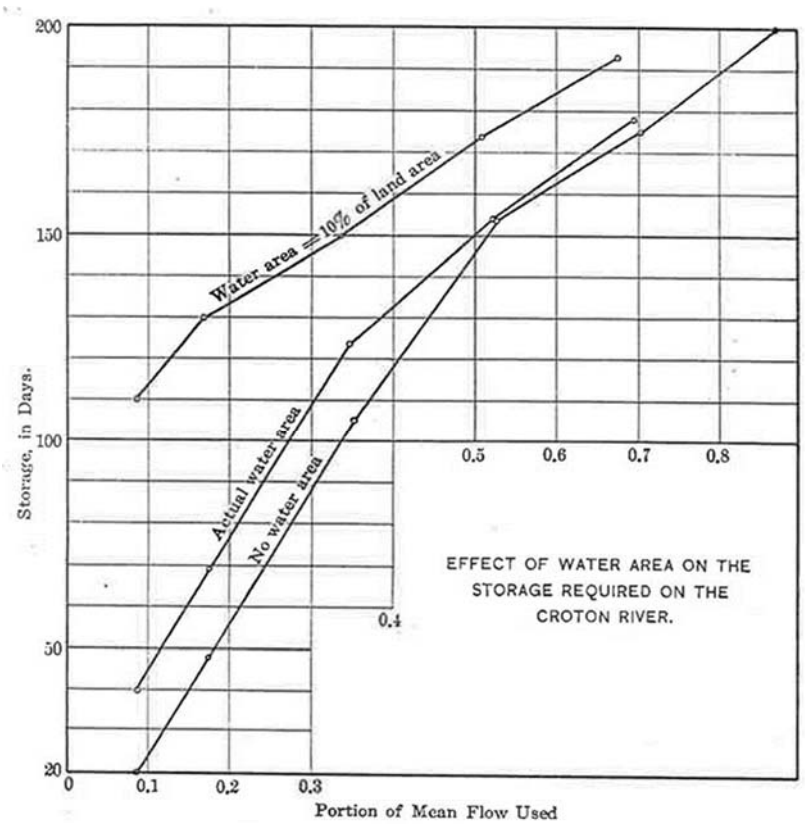


FIG. 28.

This nearly constant quantity can be estimated from the records of a relatively short term of years by finding the mean storages required to maintain one or more assumed rates of draft and subtracting these from the normal storages for the same rates of draft for an

average year. This difference (or the average of several of them) subtracted from the normal storage diagram will give a fair estimate of the storage required under various conditions for that stream, and is probably more accurate than can be made from any but the longest and most carefully kept records of flow for that particular stream.

As to THE APPLICATION of THE NORMAL LAW OF ERROR to FLOW AND STORAGE DATA

Discussion of this point has been deferred until the data represented by Figs. 2 to 21, inclusive, could be presented. A study of these diagrams gives an indication of the extent to which the data can be analyzed in this way. If the normal law of error applied strictly, the results for each series would all be found in one straight line. That, of course, is practically impossible. A reasonable approach to a straight line indicates that the data follow the law approximately. In the cases of the longest continued records, the plotted points correspond well with the lines drawn to represent normal conditions. In the records covering shorter periods there are wider fluctuations, owing to the fact that the terms of the series are not numerous enough to have filled in all the intermediate and extreme values that would be found in a long series. In general, the longer the period of record the more closely do the results permit of plotting in a direct line.

To test the matter further, two combination series of results were prepared, showing the variations in terms of the standard variation for each series, first, of the annual flows of each stream for the 300 years, shown in Plate XXXVI; second, the quantities of storage, similarly arranged required to maintain drafts of 600 000 and 800 000 gal., respectively, per square mile of land area for each of the streams for which these figures were available. A summary of the results of these two series is given in Table 10.

The figures of Table 10 are plotted in Fig. 29. The two series correspond well, that is to say, they have the same degree of skew, and a single line, therefore, represents both. No significance is attached to the fact that they have the same degree of skew. This is considered to be purely accidental. The line representing these results is a direct line, but not a straight one. Most of the results of each are below the mean; and the variations upward, though less numerous, are greater in magnitude than the variations downward. The results may be combined further, as in Table 11.

TABLE 10.

(1) Actual variation, in terms of the standard variation.	(2) 300 Terms in Series of Annual Flow.		(3) 402 Terms in Series of Required Storage.		(4) Together: 702 Terms.	
	Number below limit in Column 1.	Percentage.	Number below limit in Column 1.	Percentage.	Number below limit in Column 1.	Percentage.
− 2.5	0	....	1	0.2	1	0.14
− 2.0	0	....	3	0.7	3	0.4
− 1.5	7	2.3	15	3.7	22	3.1
− 1.0	56	18.7	68	17.0	124	17.7
− 0.5	104	34.6	138	34.3	242	34.4
Mean	163	54.3	229	57.1	392	55.8
+ 0.5	218	72.7	275	68.4	493	70.2
+ 1.0	246	82.0	327	81.3	573	81.6
+ 1.5	373	91.0	367	91.2	640	91.2
+ 2.0	288	96.0	391	97.3	679	96.7
+ 2.5	295	98.3	401	99.8	696	99.14
+ 3.0	299	99.7	402	....	701	99.86
+ 3.5	300	100.0	....	....	702	.....

TABLE 11.

Range in standard variations.	Percentage of Terms Outside Range.			
	Below.	Above.	Total.	Computed to follow normal law of error.
0	55.8	44.2	100.0	100.0
0 — 0.5C	34.4	29.8	64.2	61.8
0 — 1.0C	17.7	18.4	36.1	31.7
0 — 1.5C	3.1	8.8	11.9	13.4
0 — 2.0C	0.4	3.3	3.7	4.6
0 — 2.5C	0.14	0.86	1.00	1.26
0 — 3.0C	0.0	0.14	0.14	0.27

In Table 11 the computed and actual variations have been compared. It is seen that the range in combined upward and downward variations agrees as closely as could be expected with the range computed from the normal law of error. The variations upward and downward, taken separately, are not equal. In other words, we have to deal with what is called a skew curve on probability paper. Both ends of this skew curve seem to be nearly straight, with a connecting curve. In drawing the curves for Figs. 2 to 21, inclusive, the ratios in Table 12, obtained from the line in Fig. 29, were used. Straight lines were used to connect these points, no special significance is attached to these figures. Other figures, differing, more or less, from them, would be found from other similar data. The important

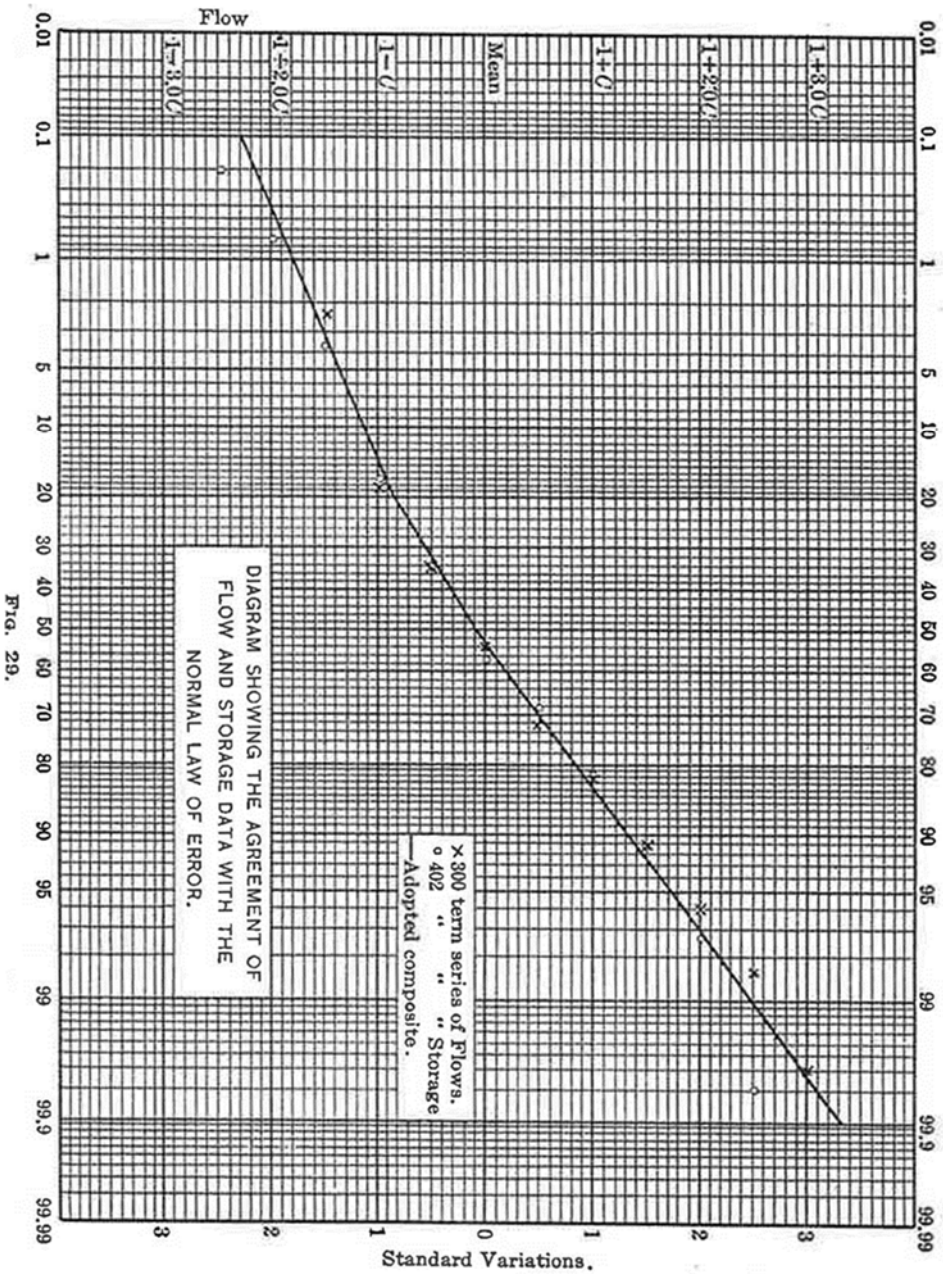


Fig. 29.

point is that data of this kind plotted on probability paper are close to a line which is direct and nearly straight at the ends, and when points are thus plotted and a line is drawn to represent them, by the graphical method herein used, it will represent the data and the probability of recurrence of certain values with as much accuracy as can be now expected.

TABLE 12.

PN'CPntage of results smalle than· limit.	ActuP.I variation, in terms of the standard val'lation.
t	- 1.sa
20	-0.88
50	-0.10
99	+2.50

Much more numerous data, covering many times longer periods, would be required to settle finally whether the law of error, as used in this way, is strictly applicable to long-term records.

It is clear that, using the normal law of error in a graphical way, with probability paper, eliminates errors growing out of the unequal variation above and below the mean, which would result from consideration of the data by arithmetical methods.

Although the evidence at hand is not conclusive that the method used is rigorously applicable to longer terms, and from the nature of the case it cannot be, it may be stated that, as far as the data go, the agreement is satisfactory, and the basis may be accepted as representing the conditions likely to occur during a long term of years with a smaller probable error than would result by any other procedure now available,

STORAGE REQUIRED to EQUALIZE VARIATIONS in ANNUAL FLOWS.

The discussion thus far has related to the storage required to equalize the flow during the months and days of any one year. With the eastern streams investigated, the annual flow will fall to 75% or less, of the mean annual flow, once in 10 years, on an average, and to 55% of the normal, or less, once in 100 years. With western streams, the range is greater. It is possible, by storage, to use more water at all times than flow in. a dry year, but to do this it is necessary to carry water over from wet years to make it available in dry ones.

In the following paragraphs "annual flow" means·the average of observations during a period of one year, stated either in millions of gallons per day, or cubic feet per second per square mile,·or as a fraction of the mean annual flow. "Mean annual flow" is the average of the annual flows for the whole record period.

On Plate XXXVI are plotted the relative annual flows of fourteen streams, the records of which have been combined into a single series of 300 years. As the mean annual flows for the several streams are different, the figures for each have been taken as the ratio of the annual flow for each year to the mean annual flow for that stream. The coefficient of variation for the annual flows has been computed separately for each stream, and the figures are entered at the top of the diagram. The records of the several streams are placed in the order of the coefficients of variation, beginning with the Hudson, which has the lowest, and ending with the South Fork of the South Platte, which has the highest.

It is obvious that the storage required to balance annual fluctuations in flow will increase with the coefficient of variation for the mean 'annual flows. It is not unlikely that the coefficient of variation may be used as a basis for measuring the storage in such a way that the results will be general, and would apply equally to all the streams within the range covered by this study.

It is believed that the top· part of the area included by the curve of flow, as plotted in the lower portion of the diagram, Plate XXXVI, which is taken broad enough to cover all the variations that occur, is the only part that needs consideration; and that the lower part, below the minimum annual flow, goes forward, in any event, with regularity through all years, and requires no storage to maintain it from year to year. Increasing or decreasing the quantity of this constant lower part has practically the effect of decreasing or increasing the coefficient of variation, but is without effect on the standard variation. With this condition in mind, the unit of storage was selected as the standard variation in annual flow. This is obtained directly from the records of annual flows, or otherwise by multiplying the mean annual flow by the coefficient of variation. The rate of draft must also be expressed in a form in which the coefficient of variation will be an element. The method selected was to compute the storage required for drafts equal to the mean annual flow,

less a certain part of a. "standard variation". These may be represented conveniently by the formula:

Mean Annual Flow X [1 - k X (Coefficient of Variation)].

Method of Calculating Annual Storage. -The calculation is carried out by assuming an indefinitely large reservoir, full at the beginning of the record period, with a constant assumed rate of draft from it, and finding the depletion, if any, at the end of the first year, with the flows occurring as shown by the records, and at the end of the second year, and of every year for the whole period. In carrying this out, monthly flows are disregarded; only mean annual flows are taken into account.

The rate of draft corresponding to the mean annual flow less 0.4, 0.6, 0.8, 1.0, 1.2, and 1.5 standard variation in annual flow, and also the value of one unit of storage, is calculated for each stream separately, and these are used in examining the terms in the series derived from the record of that stream. The required storage, that is to say, the computed depletion of an indefinitely large reservoir at the end of each year, is then stated in units of storage. When deficiency of storage is indicated at the end of any of the subdivisions of the whole series corresponding to the records of one stream, it is carried forward into the record of the following stream until the reservoir would have refilled.

The quantities of storage, computed in this way for each year in the 300-year series for each of the six assumed relative rates of draft, are shown graphically on Plate XXXVI. This plotting gives a good idea of. the periods and of the relative quantities of depletion at the several rates of draft in different parts of the whole series, but it is on too small a scale to be used as a basis of further calculation, and the actual figures from which it was made are used for that purpose.

It is interesting to note that the dry periods on the Sudbury and on the Croton River, when computed in this way, show substantially equal quantities of depletion, but that the Wachusett and Pequannock records, taken together, $how a period of depletion fully equal to these, and the Gunpowder and South Platte records, taken together, a depletion only slightly less, thus indicating, as far as these data go, that storages as great relatively as those required at the dry

times about 1880 for the Croton and Sudbury may be expected to recur from time to time in other streams,

The plottings also indicate that the method of bringing the coefficient of variation into the calculation has practically accomplished the desired purpose of arranging all the records so that the computed number of storage units do not differ, or at least do not differ widely, for streams having high coefficients of variation, and for those having low ones. In other words, the depletion for the various relative drafts fa about as high at one end of the diagram as at the other. If the adjustments were not fairly well made, there would be an excess of storage indicated at one of the ends.

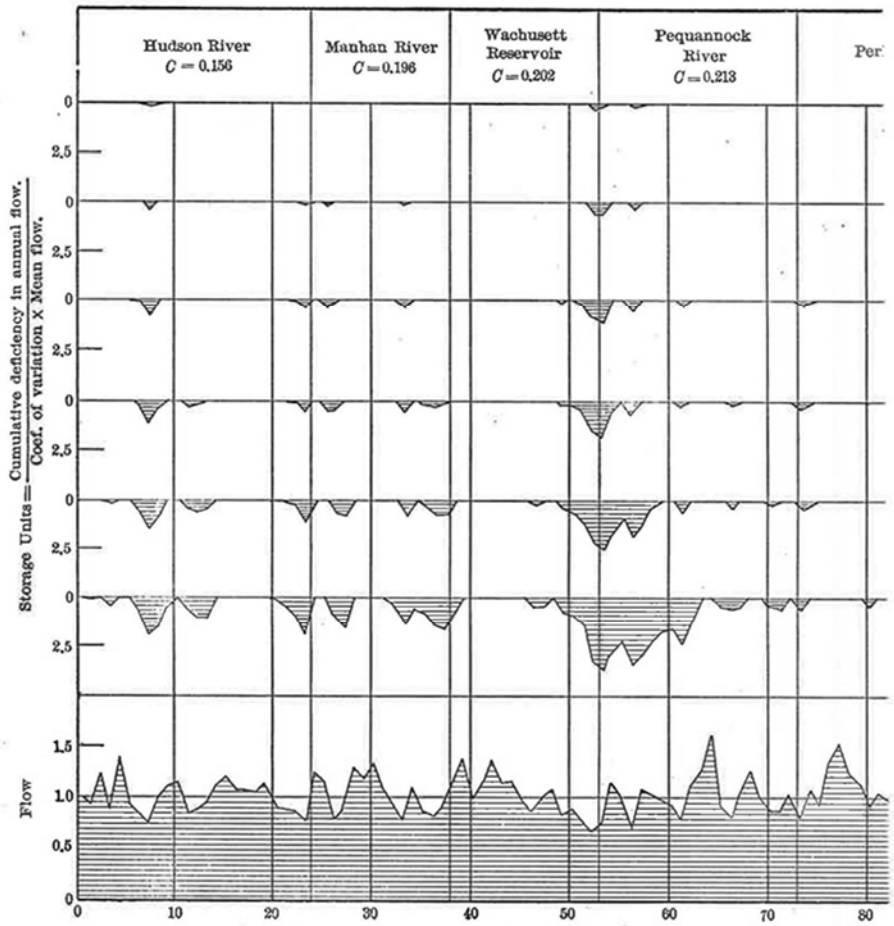
On Plate XXXVII a.re plotted, for each of the six rates of draft, the figures representing the cumulative number of storage units at the end of each of the 300 years, or as many of them. as show depletion, arranged in the order. of their magnitude, and lines have been drawn to represent, as nearly as possible, the normals for each series. This plotting is made on probability paper. The lines drawn to represent the data are straight, and the deviations of individual points: from them are small. From this diagram the values in Table.13 are taken.

TABLE 13.- ANNUAL STORAGE, IN UNITs of STANDARD VARIATION in ANNUAL FLOW, REQUIRED TO BALANCE ANNUAL FLUCTUATIONS in STREAM FLOW WITH VARIOUS RATES of DRAFT

:elative rate f draft	80% dry year	90% dry year	5%:	8	99% ry
.	1.40	2.01	2.52	8.10	8.48
	0.75	1.21	1.59	2.02	2.81
	0.30	0.65	0.95	.28·	1.4!!
	0.04	0.81	0.55	.81	0.97
		0.07	0.27	.47	0.62
			0.00	.19	0.31

In Table 14 the coefficient of variation of mean annual flow has been determined from certain other streams for which run-off records are available, for the purpose of getting a somewhat broader basis for forming a judgment of the probable coefficient of variation for other streams. No effort has been made to secure completeness in to.is table, which has only been extended to cover certain data readily available, mostly from records of the U.S. Geological Survey.

Final Arrangement of Annual Storage.-The data of Table 13 have been replotted in Fig. 30, lines being drawn for years of each of


Hudson River
C = 0.156
Manhan River
C = 0.196
Wachusett Reservoir
C = 0.202
Pequannock River
C = 0.213
Per
Storage Units = Cumulative deficiency in annual flow. / Coef. of variation x Mean flow.
Flow
0
2.5
1.5
1.0
0.5
0
10
20
30
40
50
60
70
80

the five degrees of dryness used in the calculation, showing the quantities of annual depletion to be anticipated, plotted on a base of the relative rates of draft. This diagram can be used conveniently in computing the probable annual storage required for a given stream.

TABLE 14.

Stream.	Catchment area, in square miles.	Years in record period.	Coefficient of variation in annual flow.
Kennebec River, Waterville, Me.	4 270	19	0.26
Androscoggin River, Rumford Falls, Me.	2 220	17	0.16
Cobbossecontec Pond, Gardiner, Me.	240	21	0.25
Mystic Lake, Mass.	27	20	0.28
Connecticut River, Hartford, Conn.	10 234	13	0.15
Connecticut River, Holyoke, Mass.	8 660	19	0.21
Passaic River, Dundee Dam	822	17	0.21
Susquehanna River, Harrisburg, Pa.	24 000	20	0.16
Potomac River, Point of Rocks, Md.	9 650	15	0.33
Yadkin River, Salisbury, N. C.	.....	14	0.27
Ocmulgee River, Macon, Ga.	2 425	18	0.26
Savannah River, Augusta, Ga.	7 294	17	0.25
Ohio River, Wheeling, W. Va.	23 800	22	0.19
Tennessee River, Chattanooga, Tenn.	22 418	21	0.20
Kansas River, Lawrence, Kans.	58 550	14	0.65
Republican River, Junction, Kans.	25 837	9	0.61
Arkansas River, Cañon, Colo.	3 060	19	0.23
Rio Grande, Del Norte, Colo.	1 400	16	0.34
Bear River, Collinston, Utah	6 000	11	0.23
Provo, Provo Cañon, Utah.	640	16	0.22
Mill Creek, Salt Lake City, Utah	21	13	0.35
Parley's Creek, Salt Lake City, Utah.	50	11	0.58
City Creek, Salt Lake City, Utah.	19	10	0.34
Humboldt River, Elko, Nev.	2 840	14	0.41
Tuolumne River, La Grange, Colo.	1 501	16	0.41
Columbia River, Dalles, Ore.	237 000	32	0.20
Willamette River, Albany, Ore.	4 860	17	0.26
Sweetwater Dam, California	.....	24	1.37

It may be noted that the expression, "95% dry year", as used herein, does not refer to any particular year. Its use means that, with a given rate of draft and unlimited storage, there would probably be 5 years in 100 when the depletion of storage would exceed the limit shown,

It is probable that 2 or more of the 5 years would follow consecutively in one period of several years of low average flow.

COMBINED RESULTS FROM MONTHLY AND ANNUAL STORAGE

It is now possible to construct a diagram showing the storage required to maintain various rates of draft for years of different degrees of dryness for a stream for which the mean annual flow, the coefficient of variation in annual flows, and the ground-water storage are known. Fig. 31 shows the Croton data arranged in this way, and

Fig. 32 shows the Sudbury data. Table 15 shows the. method by which the figures from which the diagrams are made were obtained.

TABLE 15.-CROTON STORAGE.-95% YEAR.

Mean annual flow, 1 sq. mile land area, 1137 000 gal. daily. Coefficient of variation in mean annual flows, 0.239.

Constant deduction for ground-water storage, etc., Croton River, as in Table 9, 23 days.

Storage unit, 1137 000 X 365 X 0.239 = 99 500 000 gal.

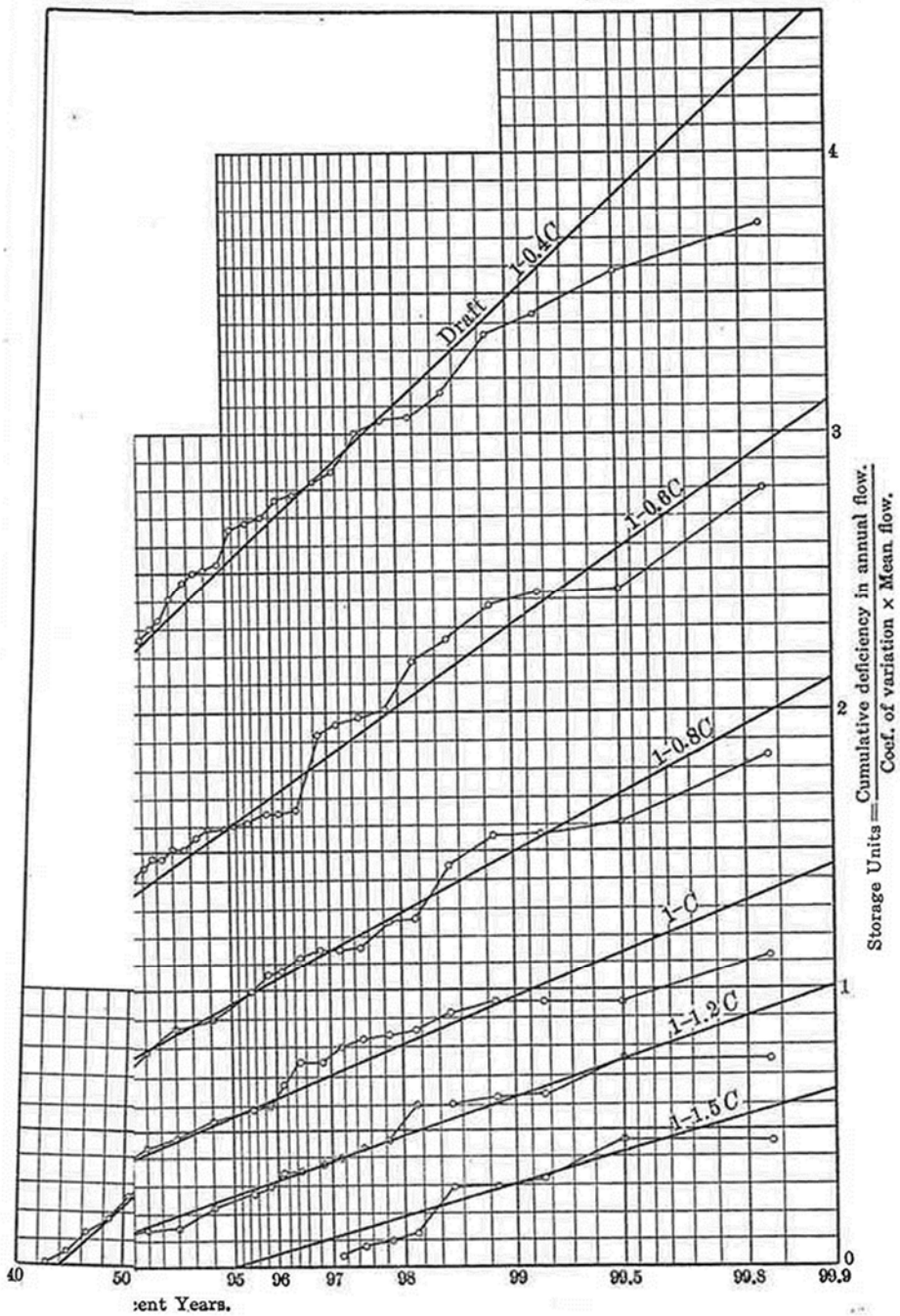
Draft, in gallons per day per square mile of land area.	Draft divided by mean flow.	Monthly and Daily Storage.			Annual Storage.			Total storage required.
		Days' storage required; normal diagram.	Days' storage less 23 for natural storage.	Storage, in millions of gallons per square mile.	Relative rate of draft.	Units of storage required.	Millions of gallons per square mile of land area.	
1 028 000	0.904	233	210	216	1 — 0.4C	2.52	251	467
973 000	0.856	227	204	198	1 — 0.6C	1.59	158	356
918 000	0.808	220	197	181	1 — 0.8C	0.95	95	276
866 000	0.761	213	190	164	1 — 1.0C	0.55	55	219
811 000	0.713	206	183	148	1 — 1.2C	0.27	27	175
729 000	0.641	193	170	124	1 — 1.5C	0.00	0	124
650 000	0.572	181	158	103	.....	.....	....	103
600 000	0.528	172	149	89	.....	.....	....	89
500 000	0.440	154	131	65	.....	.....	....	65
400 000	0.352	133	110	44	.....	.....	....	44
300 000	0.264	109	86	26	.....	.....	....	26
200 000	0.176	80	57	11	.....	.....	....	11

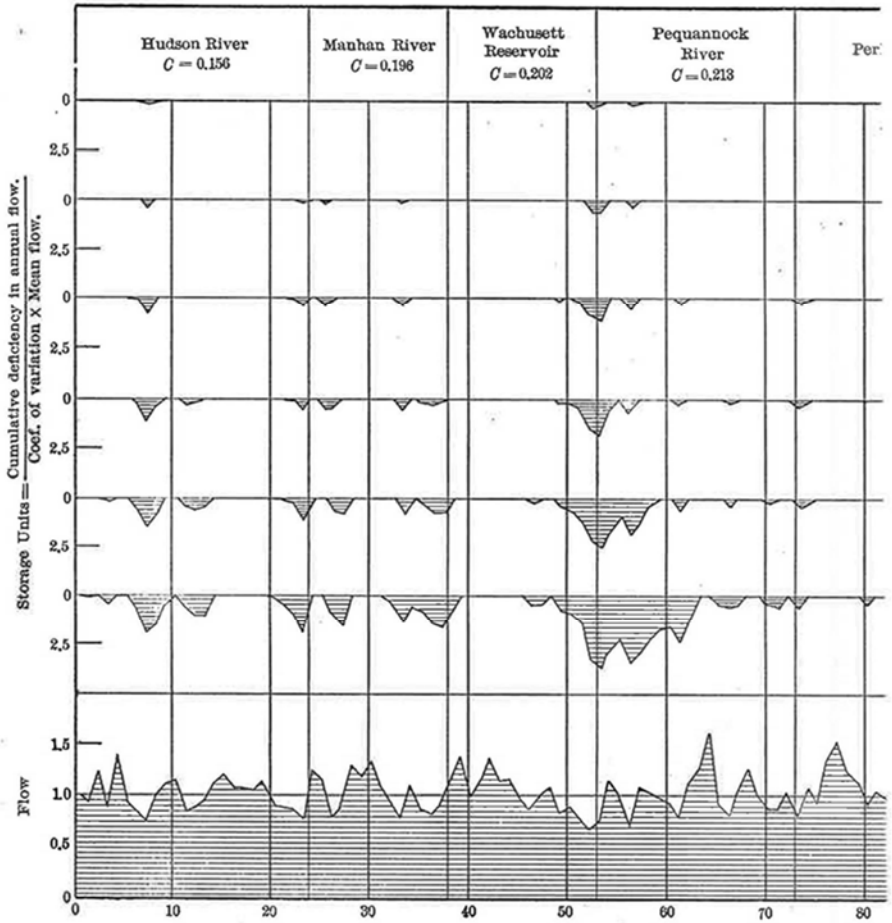
Using the diagram, Fig. 33, the construction of which will be explained subsequently, the foregoing calculation is more easily made, with the same results, as shown in Table 16.

TABLE 16.

Draft, in millions of gallons per day per square mile of land area.	Draft divided by mean flow.	Days' storage, 95%-year storage diagram.	Days' storage, less 23 for natural storage.	Days' total storage required per square mile of land area.
1.028	0.904	478	455	468
0.973	0.856	385	362	352
0.918	0.808	322	299	274
0.866	0.761	278	255	221
0.811	0.713	240	217	176
0.729	0.641	195	172	125
0.650	0.572	182	159	103
0.600	0.528	173	150	90
0.500	0.440	154	131	66
0.400	0.352	133	110	44
0.300	0.264	109	86	26

PLATE XXXVII.
TRANS. AM. SOC. CIV. ENGRS.
VOL. LXXVII, No. 1308.
HAZEN ON
STORAGE TO BE PROVIDED
IN IMPOUNDING RESERVOIRS.





On Figs. 31 and 32 two sets of lines have been drawn. The first are solid, and represent figures computed from the normal storage curve with a deduction of the constant found for that stream, which for the Croton is 23 days and for the Sudbury, 25 days. The second set, dotted lines, and the figures from which they are plotted, were scaled from Figs. 8 and 19, with the addition of 9 days for daily storage. In

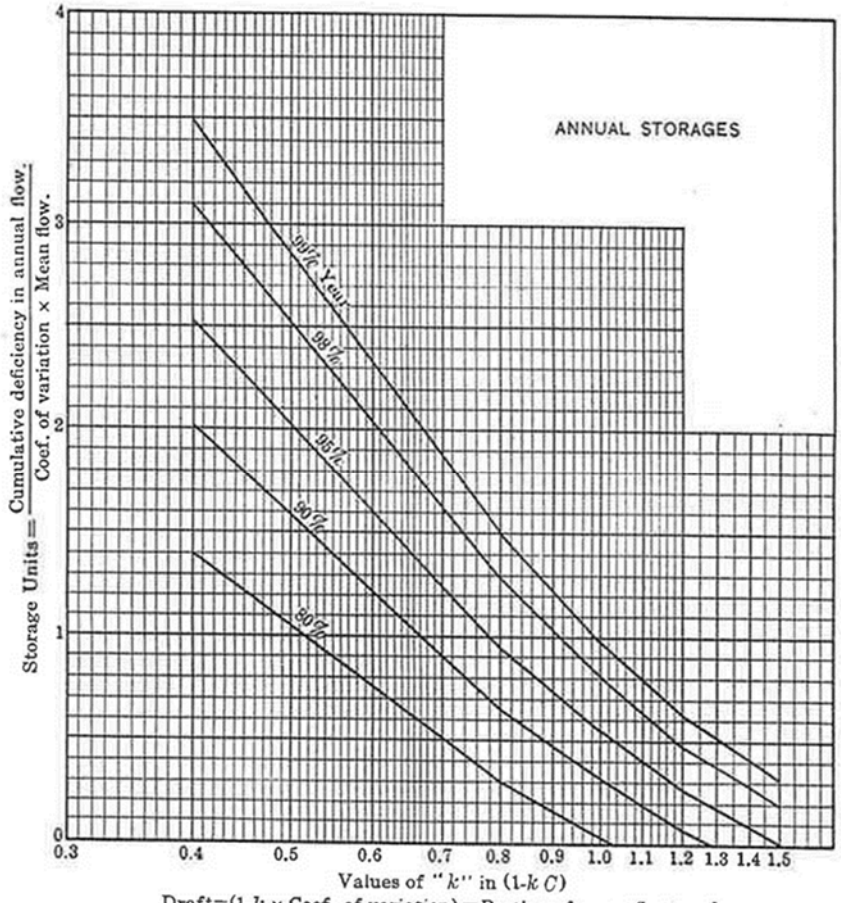


FIG. 30.

other words, they are derived solely from the records of the stream itself, and the normal diagram is not used. The allowances for annual storage were made, in all cases, from the figures in Table 13. Where the two sets of lines were practically in the same position, the dotted one was not drawn. For both the Sudbury and Croton, for the higher rates of draft, which are practically the most important ones, the annual storage

dominates, and it makes no appreciable difference which of the two methods of calculation is used. For the lower rates of draft, there are deviations. In the Croton these deviations are small in the Sudbury they are greater. In the latter the lines showing the actual records for years of different degrees of dryness are somewhat closer to one another than those deduced from the normal diagram, and with the Croton they are farther apart.

It is a matter of debate whether the lines obtained from the normal storage curve, or from the records of the stream itself, best represent the conditions on a particular stream. On the Croton, with its longer record, it makes but little difference. On the Sudbury, with a shorter record, the divergencies are greater. The writer feels that the lines obtained from the normal storage curve are more reliable than those from this relatively short-record period.

Critical Storage .-The point in the storage curve which marks the beginning of the use of storage to carry water over from one year to another is characterized by a well-marked angle. As many of the phenomena of storage differ somewhat, below and above this point, it will be convenient to give this angle a name, and it will be called the "critical point".

The values for storage found and used by Messrs. FitzGerald, Stearns, and Freeman, and stated in their several publications, are plotted on these diagrams for comparison with those calculated at this time. In a general way, for values below the critical point, they correspond to the 98% dry year; above that point they correspond to the 95% dry year. In other words; they represent conditions which would be expected to recur not oftener than once in 50 years and 20 years, respectively.

There is a particularly interesting case in connection with the Croton, where the storage, as first computed by·Mr. Freeman, for drafts of 900 000 and 1 000 000 gal. per day per sq.·mile, gave results which, as plotted on Fig. 31, come between the 90 and 95% years. Mr. Freeman showed, by an analysis of the conditions that an extraordinary rain occurred in the middle of what was otherwise the dryest period, be considered that there was no certainty of like rains occurring coincidently with other similarly dry periods, and made certain allowances on the basis that the quantity of rain that actually fell might have been less. He concluded that certain larger values

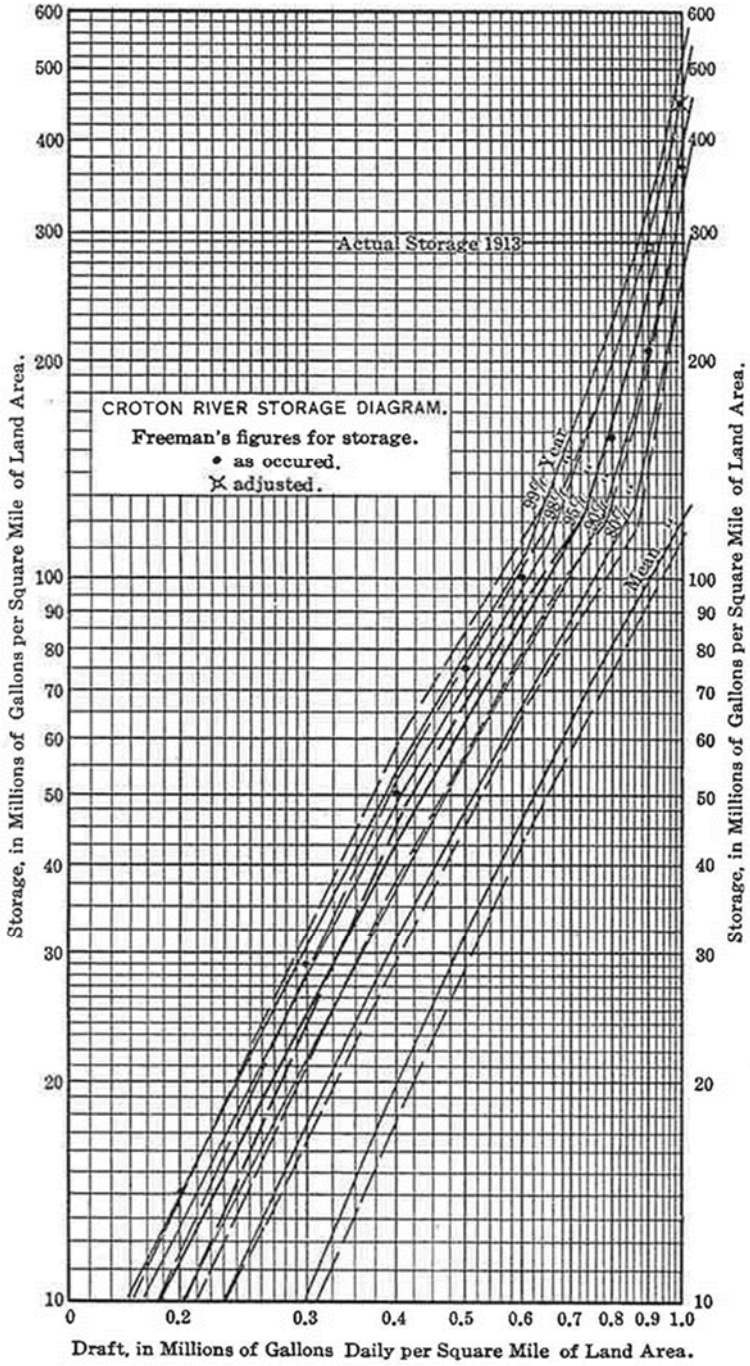

CROTON RIVER STORAGE DIAGRAM.
Freeman's figures for storage.
• as occured.
X adjusted.
Actual Storage 1913
99% Year
98%
95%
90%
80%
Mean
Storage, in Millions of Gallons per Square Mile of Land Area.
Draft, in Millions of Gallons Daily per Square Mile of Land Area.

FIG. 31.

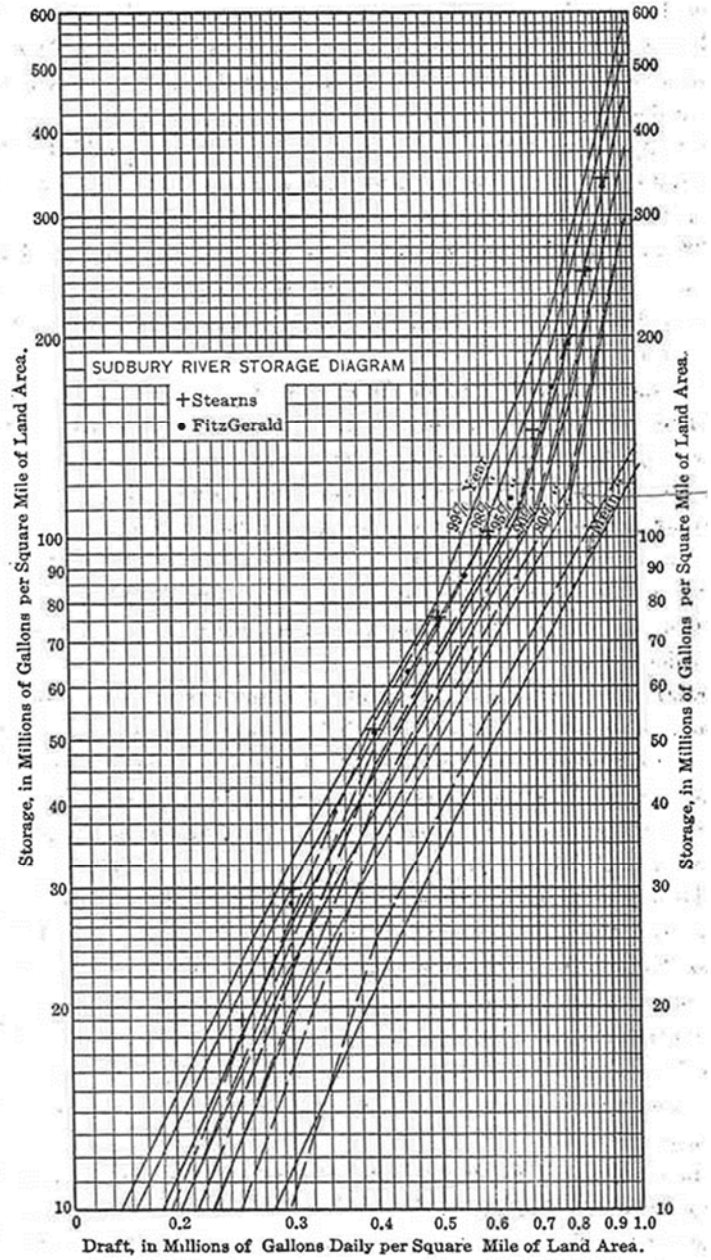

SUDBURY RIVER STORAGE DIAGRAM
+Stearns
• FitzGerald
99% Year
98%
95%
90%
80%
Mean
Storage, in Millions of Gallons per Square Mile of Land Area.
Draft, in Millions of Gallons Daily per Square Mile of Land Area.

FIG. 32.

should be used in estimating storage requirements. His revised data, as plotted in. Fig·31, fall but little below the estimate for a 98 % year, except that for draft of 800 000 gal. per day per sq. mile, which is near the 95% dry year line.

A comparison of the whole curve indicates the desirability of the corrections that Mr. Freeman made, and suggests that they might have been extended with advantage to the computed storage for a draft of 800 000 gal. per day per sq. mile. As this was not done, and as it happens to be near the point for which estimates were most frequently made in Mr. Freeman's work, the single exception is of the greatest importance.

This incident, with a recalculation which added 30% to the storage considered necessary to maintain the higher rates of draft; an addition so large that it overshadows all allowances for evaporation, ground-water storage, and other secondary conditions, shows clearly that even a 30-year period, which was the length of the Croton records at the time this calculation was made by Mr. Freeman, is entirely too short to form an adequate basis for estimating the annual and total storage required. It shows the need of a broader base of data for making such calculations.

Practically speaking, the application of the Croton data and of the Sudbury data, by the methods which have been most commonly used, and for relatively high rates of draft, have corresponded nearly with the 95% dry year by the method of calculation now used.

Some Questions as to Combining the Two Series of Results. It may be that the dryest year, from the standpoint of storage required to balance the daily and monthly fluctuations, will not also be the dryest year from the standpoint of the storage required. to balance annual fluctuations. In other words, the greatest storage in the two series of calculations may not be strictly cumulative.

An examination of the tables of results shows that the periods dryest in one series are always dry periods, if not the dryest, in the other, and the storages, therefore, are nearly, if not absolutely, cumulative.

It may be that it would be found (for example, if data were available to make the full comparison) that, in computing the combined storage for the 98% year, the storage required to balance annual fluctuations for 98% of the time should be added to the storage required

to balance monthly fluctuations up to 90% of the time, or to some other proportion than 98 per cent. On the other hand, it must be kept in mind that, in calculating the storage required to balance annual fluctuations, the average flows for the calendar year only have been considered, and, if the data were examined in more detail, periods of 365 consecutive days would be found having lower mean flows than those of any calendar year. So far as this condition exists; there would be a tendency to require somewhat more storage than is computed for the dryest years. Conditions of this kind would tend to round off the angle at the critical point in the storage curve.

The results for cumulative storage in Table 1 for the various streams were examined to see how far- they would throw light on this, and also the studies of Messrs. FitzGerald, Stearns, and Freeman, previously referred to. The data are not numerous enough to give a clear-cut indication of any tendencies that there may be at this point, and, in the absence of such indications, it may be considered that the two series of storage results are; for practical purposes, cumulative, and that the results obtained by adding one to the other are to be accepted.

Normal Diagram of Stream Flow Combined with Cumulative Storage.-It is now possible to make a diagram, combining the normal flow required to balance the daily and monthly fluctuations shown in Fig. 26 with the cumulative storage required to balance fluctuations in annual flow represented by Plate XXXVII and Fig. 30.· Such a diagram, for a 95% dry year, is presented in Fig. 33·in days' flow required to balance the daily and monthly fluctuations being the same as the 95% year line in Fig. 26, to which have been added, for all rates of draft above the respective critical points; the required annual storages for streams having coefficients of variation in annual flows ranging from 0.05 to 0.60.

The point of critical storage for each condition is indicated by the divergence of the line for that particular coefficient from the heavy diagonal base line which represents storage for all streams below their critical points.

As a matter of convenience, diagonal lines have been drawn on this diagram, showing the ratios which the required storage bears to the mean annual flows of the streams. Other lines have been drawn, to correspond with figures to be developed in a subsequent paragraph,

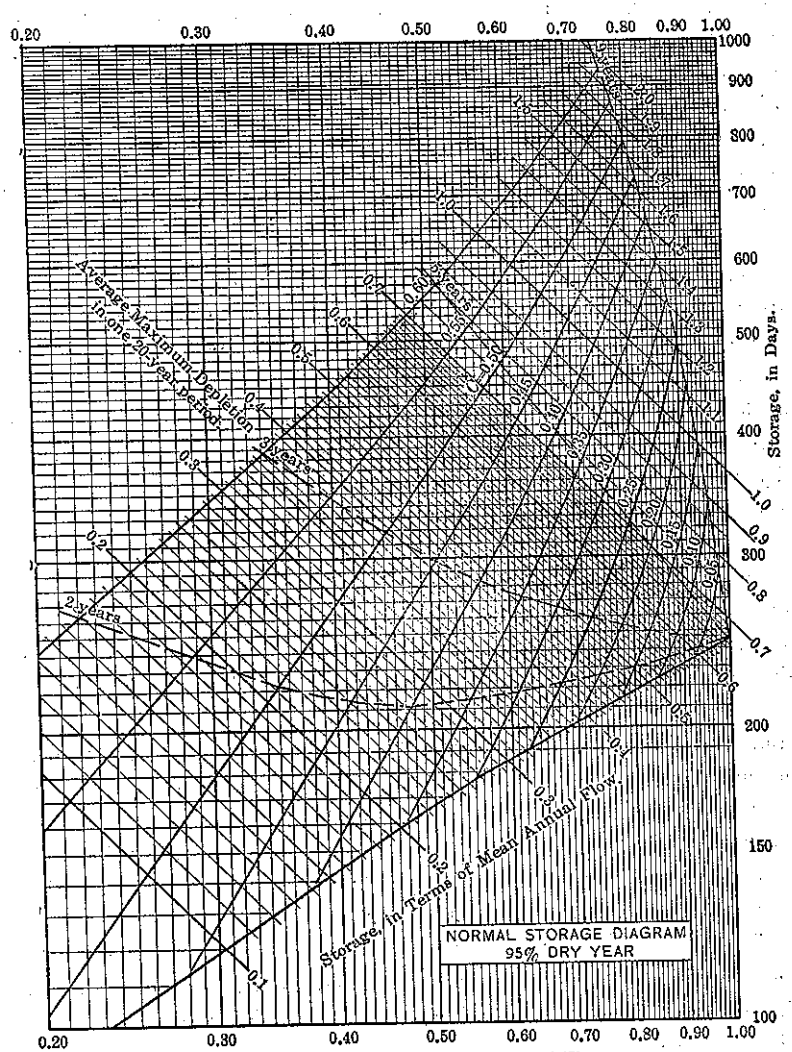

Average Maximum Depletion in one dry year period
Years
Storage, in Terms of Mean Annual Flow.
NORMAL STORAGE DIAGRAM
95% DRY YEAR
Storage, in Days.
Rate of Draft, in Terms of Mean Annual Flow.

FIG. 33.

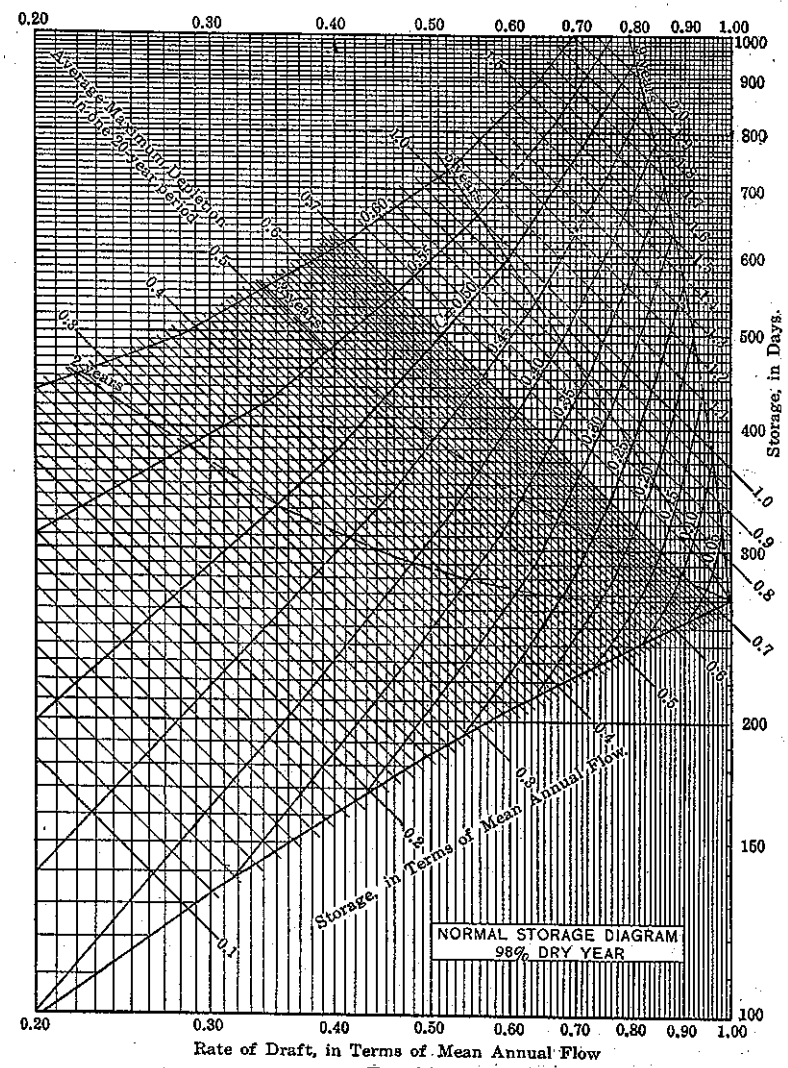

Average Maximum Depletion in one dry year period
Years
Storage, in Terms of Mean Annual Flow.
NORMAL STORAGE DIAGRAM
98% DRY YEAR
Storage, in Days.
Rate of Draft, in Terms of Mean Annual Flow

FIG. 34.

showing the points at which the reservoir will probably be less than full for periods of 2, 3, 5, and 9 years, respectively, in an average 20·year period.

Fig. 34 was prepared in the same way, but represents a 98% dry year. Similar diagrams could be prepared for 90 and 99% years, if it were desired.

The correction for natural storage and other peculiarities of the catchment area, shown for the streams for which data are available in Table 9, must be deducted from the quantities of storage taken from these diagrams.

Figs. 33 and 34 afford the most convenient means of estimating the size of reservoir required to maintain a given rate of draft, and, on the other hand, of estimating the probable draft that can be maintained from a given catchment area and storage reservoir.

APPLICATION TO DATA FOR SEVERAL STREAMS.

In Fig. 35 the foregoing data, arranged for a 95% year, have been applied to the records of the eastern streams used in this discussion, and the results have been reduced to gallons per square mile of land area of flow and storage in each case. This diagram shows, in compact form, how the storages required on different streams differ from each other; it will be found convenient for making estimates of stream flow when the record of a particular stream is to be used as a basis.

The lines have been continued at the upper end to the point where, as will be shown in a subsequent section, the reservoir will probably remain less than full for 9 years in an average 20·year period.

Rainfall.-The relation between rainfall and run-off is one that has been much discussed, and one that is not to be taken up in this paper. It may be mentioned, however, that studies of rainfall records indicate that the methods of analysis herein proposed for run-off data are also applicable to rainfall records.* The standard variation in rainfall is not very different from the standard variation in run-off for the same area in some cases, as shown by Table 17.

* Sir Alexander R. Binnie's well-known work on rainfall, in Vol. XXXIX, *Minutes of Proceedings, Inst. C. E.*, suggests this, although the data were not arranged in the way now proposed. The late George W. Rafter, M, Am. Soc. C. E. suggested the application of the method of least squares to rainfall data in the report of the New York State Engineer for 1896, with a view to deducing the probability of occurrence of certain conditions of low flow, and announced the intention to discuss in some future report the final theoretical question of methods of deducing a formula. to enable one to take the rainfall and temperature records of a long series of years and deduce therefrom the probable yearly run-off of a stream.

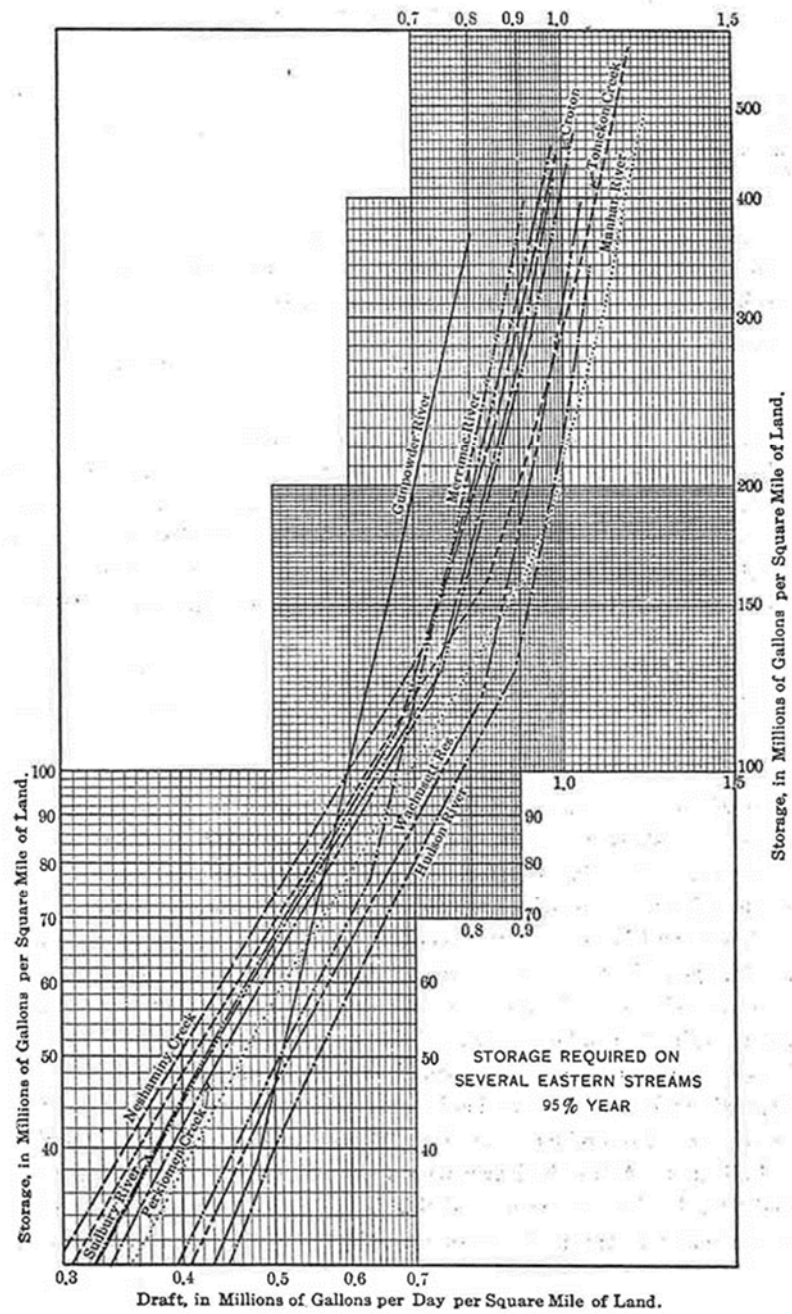


FIG. 35.

TABLE 17.—RAINFALL AND RUN-OFF COMPARED.

	Croton.	Sudbury.	Wachusett.
Mean annual rainfall	48.8 Inches	45.5 Inches	46.5 Inches
Mean annual run-off	24.0 "	22.9 "	24.0 "
Standard variation in annual rainfall	6.0 "	6.5 "	6.6 "
Standard variation in annual run-off	5.7 "	5.7 "	4.8 "

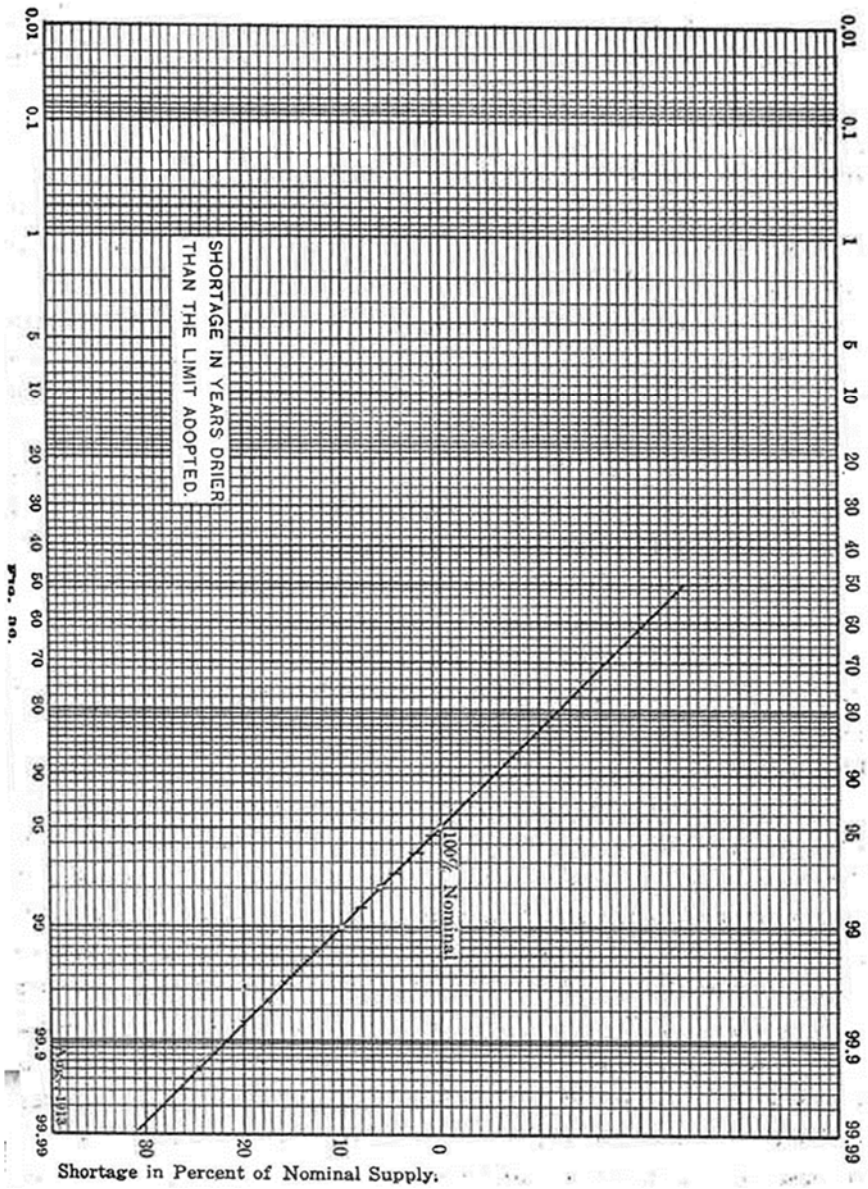
The agreement between these figures is interesting, and suggests a means of forming an idea of the probable standard variation in mean annual run-off from. rainfall records in cases where adequate stream gaugings are not available.

As to the Shortage in Years Dryer Than the Limit Adopted.-If storage is provided to maintain the flow 19 years out of 20, the question must be considered as to how much the available supply will fall below the nominal supply in the twentieth year. The twentieth years will not be alike in their degrees of dryness, and the short age of water will range from nothing to the quantity indicated as available for the same quantity of storage in the 99% year, and to still lower quantities in years that recur at longer intervals than 100 years.

The water available in the 98% year will be very nearly equal to the average quantity obtainable in all the years dryer than the 95% year.

The data for eastern streams indicate that the supply in a·98% year will be less than in a 95% year by from 3 to 9 per cent.· The percentage shortage is greatest with small storages and drafts, and least with the highest storages and drafts: As a representative (although not necessarily an average) figure, 6% may be taken.

There will be 5 years in a century with deficiency in supply when a 95% year is taken as a basis. Actual records do not suffice to show the shortages in these years. Records many times longer than those available would be required for this. Not having such records, it seems probable that the normal law of error will apply at least approximately, and that probability paper may be used for a graphical solution of this problem. In Fig, 36 the basis of such a calculation is indicated. A line is drawn passing through 100% in the 9.5% year, and 94% in the 98% year, and this is continued. Values taken from it indicate that the probable shortage in the 5 dry years of the century



will be 1, 3, 5, 8, and 14%, respectively, averaging 6 per cent. If the 98% year is taken as a basis, there will be 2 years in a century when the supply will be short, and the shortage in these years will probably be 2 and 8%, respectively, averaging 5 per cent. If, the 99% year were used as a basis, there would be 10 years in 1 000 with less than full supply, and the probable shortage for them, computed in a similar way, would be ~. 1, 2, 2, 3, 5, 6, 8, 10, and 15%, respectively, averaging 5 per cent.

These figures are to be taken as representing the probabilities of certain. shortages of water. It is to be expected that the years of shortage, with high rates of draft and storage, would. be in groups such as occurred in the Croton and Sudbury Rivers. By the methods of calculation used, the rate of draft that could be maintained by a given storage through such a group of dry years would be taken as applying to each of the dry years in that group. As a result, it would be found that the years of shortage, instead of forming a regular series, as in the figures just given for illustration, would occur in a certain number of groups, for each of which there would be a single rate of shortage. This is another illustration of the fact that much·more numerous data must be bad before the full series indicated by the normal law of error can be filled out. The figures obtained by such estimates must be regarded as the most probable ones on the basis taken, from which there are certain to be deviations. ·

As a result of the foregoing study, it appears that using the 98% dry year as a basis does not mean that the shortage in the dry year, when it finally arrives, will be materially less relatively with reference to the standard than would be the case with the 95% year or the 90% year used as a basis, but it does mean that the probability of the occurrence of such a year in any period is smaller.

The Effect of Error in Mean Annual Run-Off.-The mean annual run-off is known .only approximately, the accuracy being greater as the records on which it is based are longer. It may be important to know how much reduction in available capacity would be suffered in case the mean proved to be less than the quantity used as the basis of the calculation.

With a fixed quantity of storage, the reduction in available draft is not in proportion to the reduction in run-off; the percentage that

is utilized becomes greater as the run-off is smaller. Below the critical storage, with a fixed quantity of storage, 2% reduction in mean annual run-off results in about 1% reduction in the net available supply. Above the critical storage, 5% reduction in the mean annual run-off reduces the net available supply by about 4 per cent. These figures will vary somewhat with different assumed conditions, but they are sufficiently close for use in considering the probable effect of errors in the mean flows that are used.

As to The Accuracy of Averages of Short-Term Records.

A study was made to see whether the probable variations of short·term averages followed the rules given in the textbooks, and if not, to see how they vary from them.

The probable error of the average in a series of random data is:

Probable error in one term)/ (Square root of number of terms)

It is proposed to see whether the figures representing mean annual flows of a stream for a term of years vary as if each number were drawn out of a bag containing numbers representing all the terms of the series, or whether they occur in cycles in such a way as to modify short-term averages.

To test this point, the relative mean annual flows of the Croton River and Nesbaminy, Tohickon and Perkiomen Creeks, were selected. Each has a record of 25 years or· more, and the coefficients of variation are near together. All shorter-term records are excluded, and also the Gunpowder River, because of its higher coefficient of variation. The coefficient or variation of the whole series of 120 years is 0.235.

The figures were first divided into sixty series of 2 years each. The sixty average results form a new series, and the probable error of one term in it is ascertained. The same process was followed for other lengths of period.

The probable errors thus found were compared with those computed by the formula just given. ·

As a check on the work, the number representing each term in the 120-year series was written on a piece of paper, these pieces of paper were mixed and drawn and written do\vn as drawn, forming a new series in which the terms are identical with the terms in the

first series, but in which any tendency to cycles is presumably destroyed. This series was then divided, and the probable errors of short-term averages were determined as had been done in the first place.

The probable errors computed and found in the seven series are given in Table 18.

TABLE 18

Number of series.	Number of terms in each series.	Probable Error in Average of Each Series.		
		As computed from formula.	Series I, as occurred.	Series II, with the numbers as drawn.
120	1	15.9	15.9	15.9
60	2	11.2	12.4	11.1
40	3	9.2	10.0	9.7
30	4	7.9	10.3	8.5
24	5	7.1	8.0	5.6
20	6	6.5	8.9	5.0
15	8	5.6	7.6	5.0
12	10	5.0	5.7	4.8
10	12	4.6	4.3	4.3
6	20	3.5	5.7	4.3
4	30	2.9	3.0	2.8
3	40	2.5	1.9	2.0

The averages of periods of from 4 to 8 years have probable errors averaging 2% greater than those computed by the formula. In shorter and longer terms the excess is not more than 1 per cent. The same figures as drawn to destroy cycles, if such existed, yielded results differing from the calculated results as often in one direction as in the other. The method of calculation is thus supported, and may be accepted, except so far as it is modified by the existence of cycles in the records. The figures indicate the presence of cycles, and an appreciable, although not large, influence from them in the average accuracy of short-term averages.

Taking this influence into account, to get an average result for which the probable error will not exceed 10% with streams like those used as a basis for this study, a 4-year record is necessary. To reduce the probable error to 5%, a 12-year record is necessary, and to reduce it to 3%, 2%, and 1%, records of 28, 63, and 250 years, respectively, are required.

The probable errors of the means for the eastern streams, for the records used 'in this study, range from 2.1% for the Hudson to 3.8%

for the Gunpowder. In general, a 3% probable error for these data may be assumed. By the normal law of error, the average error is 18% greater than the probable error. Using the "probable error" as · the standard of comparison, or the measure of magnitude of actual errors, and considering the law of normal error, the following relations may be stated: There is one chance in 2 that the actual error in the mean flow will exceed the probable error; that is, the actual error is as likely to be on one side of the probable error as on the other. There is one chance in 4 that the true mean of a series· will be less than the record mean of that series by a ·quantity greater than; the probable error. There is one chance in 11 that it will be less than the record mean by more than twice the probable error; one chance in 46 that it will be less than the record mean by more than three times the probable error; and one chance in 286 that it will be less than the record mean by more· than four times · the probable error. The chance that the true mean of a series will lie above (be greater than) the record mean by a given quantity is the same as the chance that it will be below (be less than) by that same quantity, but the chance is twice as great that it will lie either above or below by the given quantity.

Probable Error in the Coefficient of Variation. The probable error in the coefficient of variation, as determined by short-term records, is also a matter of importance, as 'the coefficient of variation plays a part only second to that of the mean flow in estimating the storage required for the larger drafts with the methods now proposed.

In computing the coefficient of variation from short-term-records, it is better to divide the sum of the squares of the variations by $n-1$ instead of by n. Otherwise, the values for the coefficient decrease with the shortness of the period. With longer records, the difference is not important. In records as short as from 5·to 10 years, the difference is essential.

The average coefficient of variation for the twenty four 5-year periods previously described was found to be 0.219 as occurred, and 0.228 with the numbers drawn. These may be compared with 0.235, found for the whole series. The probable error of one 5-year determination was found to be 19.8% as occurred and 23.5% as drawn.

In other words, the probable error in the coefficient of variation deduced from one 5-year term is about 20%, and occasionally the variation will greatly exceed this.

The average coefficients of variation for the twelve 10-year periods were found to be 0.225 for the figures as occurred, and 0.230 for the -figures as drawn and the probable errors in the coefficient deduced £rom one 10·year series were 12.2 as occurred and 16.0 as drawn.

The tendency of the figures as occurred to run in cycles may be the reason for the lower variation in such figures as compared with the figures as drawn.

The probable error in the coefficient o£ variation for other periods may be assumed to vary inversely approximately as the square root of the length of the period. The figures for the periods as they occurred are preferred to those for periods as drawn, and on this basis a 17 -year record is necessary to obtain a figure with a probable error of 10%, and a 69-year record to determine the coefficient of variation with a probable error of 5 per cent.

The errors in the coefficient of variation in the data deduced for the several streams discussed at this time are thus probably within from 7 to 12% of the truth.

It is clear that short-term records must be used with caution in determining the coefficient of variation, and longer records of other streams, more or less similar to the one under discussion, may often give a better indication. of the probable value of this coefficient than short-term records of the stream itself.

An error in the coefficient of variation of the mean annual flow of a stream does not affect the required storage below the critical point: Above this point the effect can be best seen in Fig 33. For example, it appears that, with a given storage, about 5% more water is available when the coefficient of variation is 0.20 than when it is 0.25. all other conditions being equal.

Tabular Statement of Data and of the Probable Errors Therein. For convenience, the principal data with reference to the several streams are brought together in Table 19. The probable error in the mean flow and the probable error in the coefficient of variation, computed by the methods previously indicated, are shown as percentages of the total. The probable error in the deduction in days' storage,

because of natural storage, is given in days, and is found by the formula:

x = 0.67.4 (standard variation in days)/(square root of years of record)

The standard variation is taken as the value when about half of the mean flow is utilized.

TABLE 19.—SUMMARY OF DATA.

Stream.	Number of years in record.	Mean annual flow, in millions of gallons per square mile daily.	Coefficient of variation in mean annual flow.	Days to be deducted from normal storage diagram.
Bear Creek.....	11	0.188 ± 9.5%	0.471 ± 12.5%	64 ± 5
Catskill.....	17	1.400	.....	5 ± 6
Croton.....	45	1.137 ± 2.4%	0.239 ± 6.2%	23 ± 5
Gunpowder.....	29	0.911 ± 3.8%	0.308 ± 7.7%	91 ± 5
Hudson*.....	24	1.135 ± 2.1%	0.156 ± 8.5%	73 ± 4
Manhan.....	14	1.317 ± 3.5%	0.196 ± 11.1%	25 ± 8
Merrimac*.....	24	1.020 ± 3.6%	0.259 ± 8.5%	66 ± 4
Neshaminy.....	25	1.083 ± 3.0%	0.226 ± 8.3%	10 ± 6
Pequannock.....	20	1.423 ± 3.2%	0.213 ± 9.3%	.....
Perkiomen.....	25	1.080 ± 3.0%	0.221 ± 8.3%	33 ± 5
South Fork.....	11	0.053 ± 10.2%	0.505 ± 12.5%	*add* 20 ± 11
South Platte.....	8	0.075 ± 9.8%	0.412 ± 14.7%	40 ± 8
Sudbury.....	22	1.085 ± 3.6%	0.253 ± 8.8%	25 ± 5
Tohickon.....	25	1.310 ± 3.4%	0.251 ± 8.3%	2 ± 6
Wachusett.....	15	1.136 ± 3.5%	0.202 ± 10.7%	61 ± 5

*No allowance made for water area.

Probable Error in Computed Quantity of Supply, from Short-Term Records.-The errors resulting from errors in mean flow and from errors in coefficient of variation are not cumulative, because these phenomena are closely related, and the existence of a tendency to short-term cycles is an element in the results.

To determine the probable error in results obtained from short-term averages, the figures for each of the 5-year terms and the 10-year terms in the above mentioned series, were used as a basis for estimating the maintainable yield, with a storage equal to 60% of the mean annual flow for the whole term. The relative results would obviously depend somewhat on the quantity of storage assumed. Only one storage figure was used, however, this being selected to represent a reasonably complete development.

In making these estimates, allowances for ground-water and evaporation were omitted, as they would be nearly the same for each series, and in comparative results they are not important. The yield from this storage, computed from the coefficient of variation for the whole series, is 78% of the mean flow. For twenty-four 5-year periods each

considered by itself, the average computed run-off was 78%, with a probable error of 6.4 per cent.· One 5-year period (Croton 1898-1902) indicated a maintainable yield 22% above the average, and the difference as more than three times the probable error. If this were used as a basis for estimating the yield, of the Croton River, the actual yield would be less than calculated by 18 per cent.

The twelve 10-year series give 3: general average of 78.3% and a probable error for one series of 5.1 per cent. One term (Croton, 1898-1907) indicated a maintainable yield 13% above the mean, the difference being two and one-half times the probable error. The probable errors are shown in Table 20.

TABLE 20.

	In one 5-year term.	In one 10-year term.
Probable error in mean flow	8.0%	5.7%
Probable error in coefficient of variation	19.8%	12.2%
Probable error in available yield, with fixed storage, remaining the same in each series, and equal to 0.6 of the mean flow for the whole term	6.4%	5.1%

error in the calculated maintainable yield o£ an eastern stream, above the critical point, for conditions like those on the streams investigated; would be a little less than the probable error of the mean flow, and, for 10-year records and greater, may be represented approximately by the formula:

Probable error in percentage = 0.55x coefficient of variation/(square root of number of years of record)

It must be remembered there is one chance in 4 that the true result will be less than the quantity computed in a short-term record by more than the probable error; one chance in 11 that it will be less by more than twice the probable error, and one chance in 46 that it will be less by more than three times the probable error. The Croton records for 1898-1902 may be cited as indicating a deviation from the 4 5-year mean equal to more than three times the probable error for one 5-year period. Such errors may practically be guarded against in many cases by the fact that a short-term record giving such an unusually high mean, as compared with the true mean, would be certain to be a period in which the rainfall and run-off were so much above the normal that the excess would have attracted attention.

The variations in flow of different areas are such that errors up to the probable error and somewhat above. it might easily escape detection in this way; but, with errors two or, three times as great as the probable error in one short-term series, the. cases would be rare when some local evidence of unusual conditions could not be obtained by careful inquiry that would serve as a warning against placing too much dependence on the short-term average.

Cycles of Run-Off.

On Plate XXXVIII are shown the relative annual yield of the several eastern streams for each year, and also the required quantities of monthly storage to maintain drafts of 400 000, 600 000, and 800 000 gal per sq. mile of land area from each stream.

This diagram shows that periods of wet weather and dry weather extended generally over the area occupied by these.streams so that in a general way, the fluctuations are somewhat similar.

Wide local differences are shown, and these are much wider in some years than in others. Generally, the fluctuations above the mean are greater in magnitude and shorter in duration than those below the mean. One year, 1889, was notably wetter than any other covered by the records.

Only the Sudbury and Croton records go back to the dry period in the early Eighties, and they do not indicate the recurrence of other years as dry. However, there are some years, notably 1895 and 1900, and 1908, 1909, and 1910, when the required monthly storage on some of the streams was as great, or nearly as great, for some of the drafts, as was the: case in 1880 and in 1883 for the Sudbury and Croton.

The general impression obtained from inspection of the diagram is that there are cycles, and that a. median line of annual yields will be found to have a high point in 1878, swinging down to a low point in 1882, a summit in 1889, a low point in 1895, a summit in 1903 and another low point in 1910, indicating in. a general way, cycles with periods of from 10 to 12 years, but with some minor summits and low points, which, if·taken into account, would shorten the average period.

Long-Term Cycles in Run-Off. There is no direct, experimental evidence sufficient to serve as a basis in investigating the existence and magnitude of long-time cycles. There are some indirect data, especially from rainfall records. The relation between rainfall and

run-off is not exact, but the demonstration of cycles in rainfall records may be taken as evidence of the existence of like cycles in run-off records, although the percentage variation would not be expected to be the same. ·

Vermeule* gives a summary of long-term rainfall data in the United States, and states:

“A careful study of these long series shows a strong tendency to cycles of high and low precipitation. We are likely to have a period of years of high average rainfall, followed by a low period. We see at once that, under these conditions, differences shown by comparing a series of eight or ten years' length at one station with a fifty or sixty year series at another, may be entirely differences of time and not of place."

A chart at the same place shows the secular changes in annual rainfall. In the Philadelphia record, in 68 years there are eight well-defined summits, indicating a complete cycle every 8·years. The summits are not of the same magnitude, but indicate a gradual progressive increase up to about 1868, followed by a subsequent decrease.

Arthur T. Safford, t .M:. Am. Soc. C. E., states:

"It is interesting to note that the average precipitation by 10 year periods rose steadily from 1832 to between 1880 and 1890; since which time the average has begun to decrease. While we should not attach too much significance to this, the fact remains that if we ever hatv 10 continuous years with as low an average precipitation as fell during the years previous to 1840, a good many water supplies in the state will have to be enlarged.

"These differences in rainfall are so clearly marked that I will show this comparison in the form of the following table and diagram (not reproduced)

"Period Average of	Lowell.	Providence.	New Bedford.	Boston.	Average of 4 Stations.
	Inches.	Inches.	Inches.	Inches.	Inches.
9 Years, 1832-1840.....	37.98	36.87	44.27	41.02	40.03
10 " 1841 1850.....	40.27	41.21	44.94	42.88	42.32
10 " 1851-1860.....	44.14	43.73	44.76	50.09	45.68
10 " 1861-1870.....	45.93	47.56	46.27	57.24	49.25
10 " 1871-1880.....	45.24	47.84	46.88	50.26	47.55
10 " 1881-1890.....	46.47	49.07	48.69	48.48	48.18
10 " 1891-1900.....	44.46	48.89	47.72	45.46	46.63 "

* Geological Survey of New Jersey, Vol. III, 1894, p. 12.
† Report on Fall River Water Supply, July, 1902.
‡ The diagram is not reproduced.

Turneaure and Russell*, state:

"The question of a gradual change in the yearly rainfall is one the solution of which would doubtless require data covering several centuries. The rainfall for a particular locality may average considerably below the mean for many years, after which may follow, perhaps, an equally long period of surplus. In an analysis of several records extending over many years it was found that during an 83-year period at New Bedford, Massachusetts, the average for 10-year periods were as high as 16 per cent above the mean and 11 per cent. below; for 60-year periods the extremes were, at St. Louis, 17 per cent. and 13 per cent. and at Cincinnati, 20 per cent. and 17 per cent. For a 25-year period the extreme variations were 10 per cent. for both New Bedford and St. Louis#. From this it is seen that to establish a reliable mean it requires a record extending over a long period of time.

"The variations or cycles above referred to, that extend over several years, are in some cases very marked, but they are very erratic and as yet quite incapable of being predicted."

They also present a diagram in which the fluctuations of averages of precipitation records of certain localities are given. In the curve representing New England there are six clearly marked summits in the period of 60 years, or one every 10 years.

The line showing Ohio Valley stations has eight summits in a period of 60 years, or an average of one every 8 years, and the fluctuations do not correspond in point of time with those of New England. The progressive change in the general average also does not correspond. ·The total variation is less than in New England where there was a first summit about 4% above the average in 1848. The line then falls to about 4% below the average in 1872, and a second summit 2% above the average was reached in 1885.

A line for the middle Mississippi Valley shows five summits in a period of 35 years, or, on an average, one every 7 years.

Taking all these data together, there is indication of the existence of cycles with average periods of from 7 to 10 years, and with an ordinary variation in rainfall from 10 or 12% above to 8·or 10% below the mean, in the general swing of the curve, and without going to the extremes of 1 or 2 years. There is also indication of a much longer cycle with a range of not more than 4 to 6% above and below the average, and with a period of 40 to 60 years.

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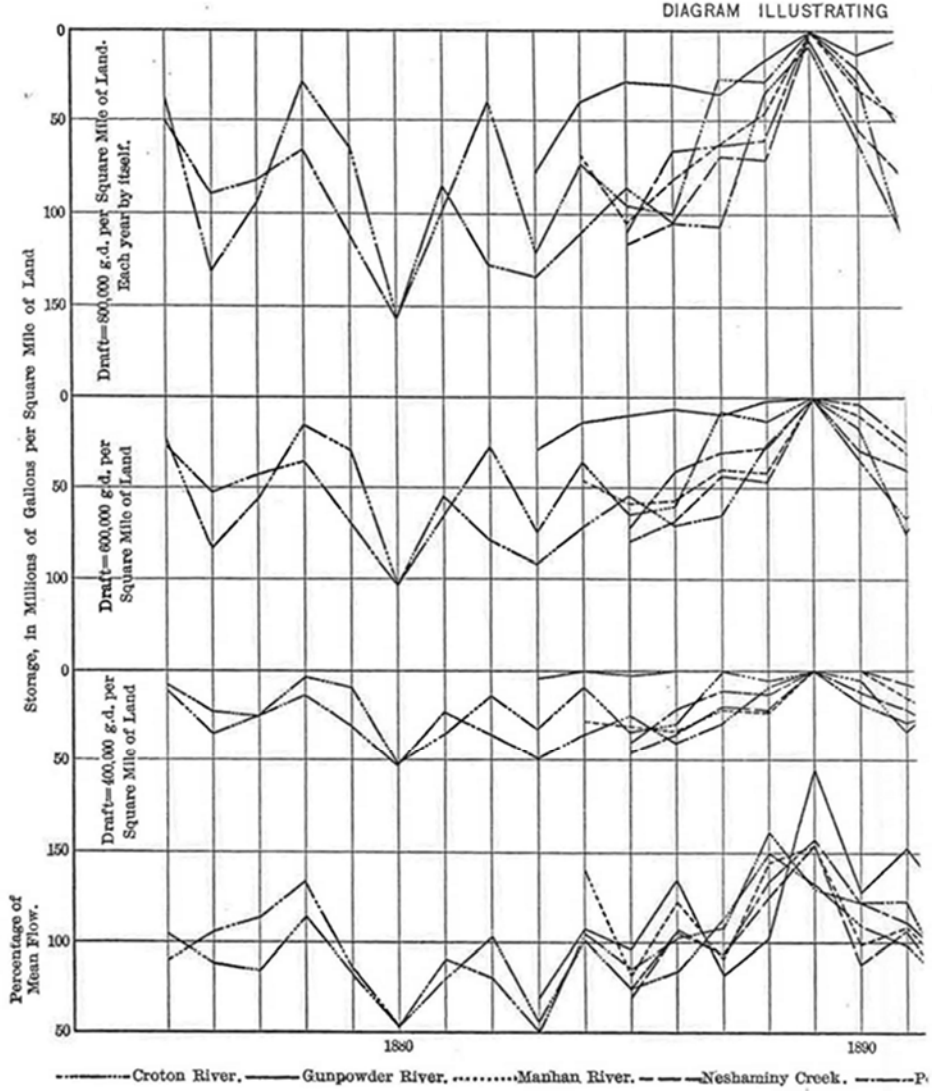
*"Public Water Supplies", p. 41.

"Bulletin D".

DIAGRAM ILLUSTRATING



Undoubtedly, there may be other longer cycles, with periods running into centuries, but no data are at hand that could be expected to show them if they exist.

As to the LENGTH of TIME THAT a RESERVOIR WILL REMAIN LESS THAN FULL

There is well-grounded objection to the use: of a development so large that the reservoir will be less than full through a considerable period of years. When a reservoir remains drawn down for a long time, vegetation grows on its shores, and much additional work must be done, as it refills, to prevent impairment of the quality of the water by the subsequent decay of the submerged vegetation. This may be less important in the future than it has been in the past, owing to the increasing use of methods of treating and purifying water which will result in the removal from it of deleterious substances resulting from such growths and decompositions. However, the matter remains one of considerable importance, and should not be lost sight of in any study for a complete development.

Methods.-The data from which Plate XXXVI was prepared were used and the number of years that the annual flow is less than the corresponding draft and the number of winters during which the reservoir will not refill were counted and the percentages obtained.

From the data referred to above we find for a draft of 1- 0.4, 27 periods in a total of 252 years, varying in length from 3 to 15 years, during which the reservoir is less than full. These are entered in Column 2 of Table 21 in the order of their occurrence and in Column 3 in their order of magnitude.

If the maximum period of depletion is to be computed for a 100-year period, then the 252 years in the whole series will form 2.52 such periods; and the first two terms in Column 3 and 0.52 of the third term are added to make the sum of 32.7 years for depletion for 2.52 such 100-year series. For one 100-year series the average maximum period of depletion is found by division to be 32.7/2.52 = 13 years. In a similar way, there are 8.4 terms of 30 years each, and the sum of eight first terms and 0.4 of the ninth term is 87.2, and the average maximum term of depletion in each 30-year series, 10.4 years. Similar calculations are made for 20 years and for 10 years.

TABLE 21.

Number of term. (1)	Length of depletion period, in years: $d_1\ d_2 \ldots$ (2)	Figures in Column 2 in order of magnitude: $d_1\ d_2 \ldots$ (3)	Average maximum period of depletion: D. (4)	Years in period for which D is to be found: P. (5)	$\frac{n}{P}$ = number of terms in summation in Column 3. (6)
1	3	15			
2	8	12	32.7 ÷ 2.52 = 13.0	100	2.52
3	3	11			
4	15	10			
5	4	10			
6	5	9			
7	9	9			
8	6	8	87.2 ÷ 8.4 = 10.4	30	8.4
9	4	8			
10	8	7			
11	3	7			
12	9	6	115.6 ÷ 12.6 = 9.2	20	12.6
13	11	6			
14	4	6			
15	3	5			
16	5	5			
17	7	4			
18	3	4			
19	6	4			
20	10	4			
21	7	3			
22	12	3			
23	3	3			
24	10	3			
25	4	3	165.6 ÷ 25.2 = 6.6	10	25.2
26	3	3			
27	6	3			

Similar tables were made for all the other rates of draft represented in Plate XXXVI, and the values obtained plotted, from which smooth curves were drawn, and from these curves values of the average maximum periods of depletion were found, as shown in Table 22.

TABLE 22.-DEPLETION.

Relative rate of draft.	Percentage of years that annual flow is less than the corresponding rate of draft,	Percentage of winters when the reservoir will not refill.	Average maximum period of depletion in one 20 year period.
1-0.4C	39	58	9 years
1-0.6C	32	48	7 "
1-0.8C	24	80	5 "
1-1.0C	17	21	3.5 "
1- 1.2C	10	12	2.2 "
1- 1.5C	3	3	1.5 "

In a 10-year period, the probable maximum period of depletion is about 30% shorter than in a 20-year period; and in a 100-year period, 60% longer.

In using Table 22 it must be borne in mind that periods of depletion as long as those stated will not occur in every 20-year period. The figure given is rather the average length of the longest term of depletion to be expected in each of a considerable number of 20-year periods.

TABLE 23.-RATES OF DRAFT FOR SEVERAL STREAMS WITH THE STATED AVERAGE MAXIMUM PERIOD of DEPLETION IN EACH 20-YEAR Period.

Stream.	Percentage of water area.	Mean annual flow, in millions of gallons per square mile of land area.	Coefficient of variation.	Years of Continuous Depletion on an Average in Each 20 Year Period.			
				9 years (1-0.4C).	5 years (1-0.8C).	3 years (1-1.1C).	2 years (1-1.3C).
Hudson.....	....	1.135*	0.156	1.064	0.993	0.940	0.905
Manhan.....	0	1.317	0.196	1.214	1.111	1.035	0.983
Wachusett.....	4.16	1.136	0.202	1.044	0.952	0.883	0.836
Pequannock....	4.15	1.423	0.213	1.301	1.182	1.090	1.030
Perkiomen.....	0.00	1.080	0.221	0.985	0.890	0.818	0.770
Neshaminy.....	0.00	1.083	0.226	0.985	0.887	0.813	0.765
Croton.....	2.75	1.137	0.239	1.028	0.920	0.838	0.788
Tohicken.....	0	1.310	0.251	1.178	1.048	0.949	0.882
Sudbury.....	3.08	1.085	0.253	0.975	0.865	0.782	0.727
Merrimac.....	2.65	1.020*	0.259	0.914	0.809	0.730	0.677
Gunpowder.....	0	0.911	0.308	0.800	0.687	0.602	0.546

* Land and water area.

As to evaporation from water surface and the allowance therefor.

Notwithstanding the historic researches of Mr. FitzGerald, on evaporation, and the data which have accumulated in all the years since his results were published, the actual knowledge of the quantity of evaporation from wate1• surfaces is meager. Primarily, this is because there is no certain way of measuring the evaporation from an actual reservoir. In the best planned experiments, conditions differ rather widely *from* those of water in an open reservoir. The quantity of experimental data, also, is not great. or continuous, and is far from adequate for giving a general idea of the allowances that should ·be made all over the country.

Mr. FitzGerald's experiments at Chestnut Hill gave a mean annual evaporation of 39.1 in.

At the Lawrence Experiment Station, Massachusetts State Board of Health, H. F. Mills, Hon. M. Am. Soc. C. E., found that the evaporation from water in a wooden tub, 17 ft. in diameter, through a period of 3 years, averaged 29.2 in.

Experiments at Fall River, Mass., reported by Mr. A. 'J:'. Safford, in a tank 5 ft. 11 in. in diameter and, 3 ft. high, in a coffer-dam; observed for 3 years, with a few winter months missing, indicated a mean annual evaporation of 38.7 in.

At Rochester, N.Y., E. A. Fisher, M. Am, Soc. C.E. found that the evaporation from water in a· tub. floated on the surface of the Mount Hope Reservoir from 1891-1909, averaged 34 in.·per annum. A similar tub on land, with some results missing, indicated 17 in. per annum more evaporation than from the floating tub. This wide divergence indicates the great difference in the results from a slight change in exposure. In summer the water in the tubs was warmer than the surface water in the reservoir by 3 or 4 degrees. It is very likely that the actual evaporation from the reservoir is less than is indicated by the records from the floating tub. When the difference between the conditions in the best arranged exposure and those in actual reservoirs are considered, too much weight must not be attached even to the best experimental results.

Comparison of Rainfall Records with, Estimated, amount of Evaporation -Mr. F.P. Stearns gives the following table*.

'Table Showing Relation of Evaporation to Rainfall.

Note: +indicates excess or rainfall -Indicates deficiency.

" Month.	Average Year.			Year of Low Rainfall		
	Rainfall.	Evaporation.	Excess or deficiency of rainfall.	Rainfall.	Evaporation.	Excess or deficiency of rainfall.
	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
January.....	4.18	0.98	+3.20	2.81	0.98	+1.83
February....	4.06	1.01	+3.05	3.86	1.01	+2.85
March.....	4.58	1.45	+3.13	1.78	1.45	+0.33
April.....	3.32	2.39	+0.93	1.85	2.39	−0.54
May.....	3.20	3.82	−0.62	4.18	3.82	+0.36
June.....	2.99	5.34	−2.35	2.40	5.34	−2.94
July.....	3.78	6.21	−2.43	2.68	6.21	−3.53
August.....	4.23	5.97	−1.74	0.74	5.97	−5.23
September...	3.23	4.86	−1.63	1.52	4.86	−3.34
October.....	4.41	3.47	+0.94	5.60	3.47	+2.13
November...	4.11	2.24	+1.87	1.81	2.24	−0.43
December...	3.71	1.38	+2.33	3.55	1.38	+2.17
	45.80	39.12	+6.68	22.78	39.12	−6.34"

* Report, Massachusetts State Board of Health,1890, p. 345.

Mr. Stearns states:

"It will be-seen from the facts presented that the monthly rainfall varies much less during the year than the evaporation; also that in an average year the rainfall is 6.68 inches greater than the evaporation. The average year may be divided into two periods, one extending from May to September, inclusive, in which the evaporation is 8.77 inches greater than the rainfall; and the other extending from October to April, inclusive, in which the rainfall exceeds the evaporation by 15.45 inches.

"In the year of low rainfall the evaporation was 6.34 inches greater than the rainfall. During the warmer months, from April to September, inclusive, the excess of evaporation was 15.22 inches, and during the other six months the rainfall was 8.88 inches in excess of the evaporation. These figures indicate that a pond will. not. lower by evaporation in a dry summer more than about fifteen inches, even if it receives no water from its water-shed."

The following analysis is presented of the figures used by Messrs. FitzGerald,* Stearns, and Freeman. The first two publications refer to the Sudbury River, and the data used were the same Mr. Stearns also used Mr. FitzGerald's data on evaporation as a basis for his calculations. The only differences are in the method of applying the data. Mr. FitzGerald based his calculation on the total area, including water, and Mr. Stearns reduced the figures for flow at the outset to a basis of land area, as is done in this paper. In using Mr. FitzGerald's figures, they have been changed by the necessary calculation to make them comparable with the others. Mr. Freeman's figures relate to the Croton River, and those finally reached are in the shape of a diagram from which the values in the following tables have been scaled, with such calculation as necessary to bring them to the uniform basis. Mr. Freeman also used Mr. FitzGerald's evaporation results as a basis for his allowances.

All the figures in the following tables, therefore, rest on Mr. FitzGerald's work. They are of interest as showing the amount of the corrections actually used, and they are also important because these tables and diagrams have been those that have been used most widely in estimating the capacity of other sources.

The storage, in millions of gallons per square mile of land area, computed for no water area and for 6% water area (6.38% of the land area) are given in Table 24.

+Transactíons Am. Soc. C. 8., Vol. XXVU, p. 253.
lReport, Massachusetts State Board of Health, 1890, p. &35.
tReport upon New York Water Supply 1900, p. 23Cr'

TABLE 24.

Draft, in gallons per day.	FitzGerald. Sudbury River.			Stearns. Sudbury River.			Freeman. Croton River.		
	No water.	6% water.	Difference.	No water.	6% water.	Difference.	No water.	6% water.	Difference.
100 000	0.3	6.9	6.6	0.6	8.8	8.2	5	8	3
200 000	8.8	15.6	6.8	9.4	18.0	8.6	14	20	6
300 000	28.5	35.1	6.6	29.8	36.1	6.3	29	35	6
400 000	51.3	56.6	5.3	52.0	57.5	5.5	51	57	6
500 000	75.8	79.1	3.3	76.5	80.3	3.8	75	82	7
600 000	100.6	107.0	6.4	102.0	107.1	5.1	100	108	8
700 000	140.0	157.0	17.0	144.4	161.6	17.2	129	134	5
800 000	199.1	215.0	15.9	202.3	219.5	17.2	157	162	5
900 000	334.1	342.0	7.9	346.2	352.2	6.0	207	216	9

The figures under the headings "Difference" indicate the additional storage required to compensate for loss by evaporation with 6% water area. These may be conveniently stated in terms of the depth of water over the area of the water surface. The figures reduced to this basin are given in Table 25.

TABLE 25.

Draft, in gallons per day.	Excess Storage in Depth over Water Area, in Inches.		
	FitzGerald. Sudbury River.	Stearns. Sudbury River.	Freeman. Croton River.
100 000	6.0	7.4	2.7
200 000	6.1	7.7	5.4
300 000	6.0	5.7	5.4
400 000	4.8	5.0	5.4
500 000	3.0	3.4	6.3
600 000	5.7	4.6	7.2
700 000	15.3	15.5	4.5
800 000	14.3	15.5	4.5
900 000	7.1	5.4	8.1
Average	7.6	7.8	5.5

All this work has been checked by the writer by an independent calculation, which indicates the substantial accuracy of the methods of calculation which are thus summarized. This was done by making tables of corrected flows for the Croton, Sudbury, and Wachusett records. The actual water area was taken into account for each month, the quantity of rainfall on it and the quantity lost by evaporation, using Mr. FitzGerald's figures for all these sources. In this way the calculated run-off was obtained for an area with no water

exposed to evaporation. By similar procedure, the quantity of run-off was calculated from the assumption that there was 0.1 sq. mile of water area for each square mile of land.· The storage required to maintain certain drafts was then computed. These results are shown graphically in Figs. 2, 3, 4, 7, 8, 9, 18, 19, and 20. The excess storage, expressed in inches of depth, for an area having 0.1 sq. mile of water area for 1 sq. mile of land area, as compared with no water area, was found to be practically the same for assumed drafts of 200 000, 400 000, 600 000, and 800 000 gal. per sq. mile. For the Croton, the average excess storage required in years of different degrees of dryness was as follows:

80%	Year	6.7	Inches.
90%	"	7.7	"
95%	"	8.6	"
98%	"	9.5	"
99%	"	10.0	"

The figures are two-thirds of the maximum excess of summer evaporation over rainfall for years of the same degree of dryness. The excess storage required on the Wachusett Reservoir was slightly less than for. the Croton, and on the Sudbury slightly more. These differences result from changes in the distribution of the rainfall as much as from variations in the total quantity. It would. be easy to introduce refinements in this calculation and extend. it, but the accuracy of the data do not warrant it.

Independent Study as to Evaporation.- All the figures thus far given are based indirectly on Mr. FitzGerald's experiments. In the case of three of the streams for which flow data are available, namely, the Croton and Sudbury Rivers and the Wachusett Reservoir, there has been an increase of water area during the period covered by the records of the excess of evaporation over rainfall on this added water area was large, it would be expected that the records of flow would show the influence of this increase. To investigate this matter, each record was divided into two parts, the latter of which in each case represents a larger percentage of water area. The storages required to maintain flows of 100 000 and 200 000 gal. per sq. mile were taken, and the storage required to maintain the flow in the 95% year found for them. The results are shown in Table 26.

TABLE 26.

	Croton		Sudbury		Wachusett	
	1st series.	2d series.	1st series.	2d series.	1st series.	2d series.
Length of time, in years	24	20	10	12	7	8
Percentage of water area	1.92	3.72	2.56	3.58	2.53	3.85
Storage, in millions of gallons per square mile of land, required to maintain a flow of 0.1 million gallons per square mile of land in 95% year	3.0	3.0	2.0	1.5	0	1.5
Storage, in millions of gallons per square mile of land, required to maintain a flow of 0.2 million gallons per square mile of land in 95% year	10.7	12.1	12.3	8.3	2.4	9.4
Increase in water area	1.80%		0.97%		2.05%	
Increase in storage, 0.1	0.0		− 0.5		1.5	
0.2	1.4		− 4.7		7.0	
Average	0.7		− 2.6		4.2	
Equivalent depth of water over additional exposed area, in inches	2.8		− 15.4		6.7	
Weighting factor	44 × 1.80 = 79		22 × 0.97 = 21		18 × 2.05 = 34	

Weighted average for all three, net loss, 1.4 in.

On the basis of the weighted average; a net loss by evaporation of in. is shown. The relative loss would obviously be greater in a dryer year. Repeating the calculation for the 98% year, the excess evaporation on a weighted average is found to be 2 in. This is less than one-third of the average quantity deduced in the preceding calculations.

In the Sudbury records the second series clearly represented a less dry time than the earlier series, and a change in this respect was more than enough to offset any influence exerted by evaporation on, the increased water surface, In the same way, the second Wachusett series represents a dryer period than the earlier one, and the effect of evaporation is probably less than is indicated. by these figures. As far as these data go, they indicate that the allowances made. by Messrs. FitzGerald, Stearns, and Freeman were at least sufficient for these streams.

Obviously, the amount of this allowance, which is relatively small in these. particular cases, and not a matter of the first importance in calculating the storage on these streams, would become greater with smaller rainfall or with greater relative evaporation such as must be anticipated in many places. On the other hand, for some. reservoirs,

even in the dryest years, the rainfall on the water area will exceed the evaporation.

As to the Method of Applying the Allowance for Evaporation.- The whole process may be conveniently divided into three parts. The first consists in excluding the water area from the computation and reckoning the discharge on the land area only. This was done at the outset with most of the data used in this paper.

The second consists in finding the probable mean yield or mean loss from the water area, and adding it to or subtracting it from the estimated run-off from the land area, thereby obtaining a corrected estimate of the mean annual flow from the whole area. It may be noted that the relative variation in yield from water area is greater than in yield from land area. It follows that, as the water area increases the coefficient of variation of the mean annual flow will increase. With large percentages of water area, this is a matter that must be taken into account. With only such variations. as are represented by forming reservoirs for the development of ordinary catchment areas, the change in the coefficient will not be large, and its effect may usually be overlooked.

The third step is to find the amount by which the evaporation will exceed the rainfall during the period of depletion of the reservoir in a dry year. The amount of this excess, in inches over the whole water area of the reservoirs, represents a quantity of water which must be added to the calculated storage, in order to obtain the total required storage to maintain a given draft, or, in the reverse. calculation, it must be deducted from the capacity of a reservoir before calculation, it must be deducted from the capacity of a reservoir before calculating the maintainable yield from it. In the case of the Croton, Sudbury, and Wachusett Reservoirs, on the best available evidence, the amount of this allowance should be between 6 and 10 in. This is considerably less than the maximum evaporation in excess of rainfall in all the dry months of a dry year.

This latter, from the best data, amounts to about 15 in. The allowance, therefore, is from one-half to two-thirds of the maximum amount that a reservoir without inflow or outflow would lower in a dry year The reason the allowance is less than the full amount of such lowering is that the period of depletion due to draft of water at a steady rate does not coincide with the period during which the evaporation is greater than the rainfall.

For other climatic conditions, it is apparent that the allowance would vary. It will increase with the evaporation and will decrease with the rainfall, and it will increase relatively with the length of the period of depletion.; That is to say with a well-marked seasonal fluctuation in rainfall, the connection to evaporation may be greater than with well-distributed rainfall.

For roughly approximate work, the writer suggests that the allowance for evaporation in a 95% dry year may be estimated. as equal to the mean annual evaporation, less two thirds of the mean annual rainfall. Such a rule cannot be regarded as close, and it is probably that additional data will led to modification. On present information, the probable error involved by its use will not be very great, except in the case of large and shallow reservoirs.

Storage On a Tributary

The ordinary condition of storage is in a reservoir on a main stream, so that all the catchment area is tributary to it. It frequently happens that the best reservoir site, or the actual reservoir, is on a branch stream where only part of the catchment area is tributary to it, and the question arises as to how far the storage on such a tributary is equivalent, or nearly equivalent, to storage on the main stream.

This matter was investigated at Springfield in connection with the Borden Brook Reservoir, which, as a first development, had only one-sixth of the Littre River catchment area tributary to it. It was also investigated by Mr. R. R. Marsden and the late Richard Hazen, Jun. Am. Soc. C. E., as a graduation thesis at the Thayer School of Civil Engineering, in 1909.

Storage on a tributary is obviously as useful as storage on a main stream, up to the point where the reservoir would be certain to fill in the dryest winters. For streams of the character of those in the East now under discussion, this is represented by about 12 in. of storage for the area back of the reservoir.

Additional investigation, however, has shown that a larger reservoir is filled in the years that precede the dryest year; that it is used in maintaining the flow during the dryest year; and though it may not refill in the winter following the driest year, that fact is immaterial because enough water will be collected to serve during the year

following the dryest year; and, in the years that follow, the reservoir will be refilled before another extremely dry year occurs. It was found that with 12 in. of storage on the main stream as the greatest quantity which could be counted on to be refilled with certainty each winter, a reservoir on a tributary holding 18 in. from the area back of it, or 50% more than the maximum size that could be refilled every winter with certainty, was practically as serviceable as an equal volume of storage on the main stream. That is to say, on a given catchment area 1 000 000 000 gal. of storage on a tributary is substantially as useful as 1 000 000 000 gal. of storage on the main stream, up to the point where the area tributary to the reservoir will suffice in the dryest winters to furnish two-thirds of the capacity of the reservoir. With a larger reservoir, there is some further gain with increasing size, but in a diminishing ratio.

How Far Will it Pay to Go in Providing Storage?

In the past the question has been mainly discussed. on the basis of providing storage sufficient to maintain the supply through a period of years as dry as those taken as the limiting condition, usually those from 1879 to 1883. We now have data to make an estimate of the probability of the occurrence of years of various degrees of dryness, and it remains to determine which shall be used. Obviously, it is necessary to provide storage sufficient to maintain the supply in as dry years as any that occur with considerable frequency. It is equally obvious that it will not ordinarily pay to provide storage to maintain the full supply through a year so dry that it will recur, say, only once in 100 years. The chances are that a supply now built will have passed through the whole period during which it is able to maintain its intended service, and will have been reinforced with other supplies before the dry year for which it is built would occur.

It has not been infrequent, in American practice, for cities to have somewhat less than a full supply in very dry years, In some cases, where water is being wasted, a moderate shortage may be a blessing in disguise, for it brings about a study of conditions of supply and a stoppage of waste, which is advantageous to the system. This proved to be the case with the recent dry period in the New York city works. No such advantage results

where the services are all metered and leaks are reduced to a minimum, but on the other hand, there is inconvenience and loss from a shortage of water. Nevertheless, it is possible, in a very dry year, to cut off some uses of water and reduce the output, and it will be better to do this once in a long term of years than to spend additional money on storage to be but seldom used.

It has been pointed out that, normally, the consumption of water by a city is steadily increasing, and that the supply available at the reservoir fluctuated from year to year with the rainfall. There is only a moderate probability of a very dry year occurring at the same time that the city reaches the dry-weather capacity of the source. In other words, if the supply is designed so that the full service can be maintained 19 years out of 20, there is only a slight probability that the very dry year which cannot maintain the supply will be one of the same in which the consumption has grown until it has reached the full capacity of the source.

Another consideration that may be taken into account in cases where several systems are near each other is the possibility of buying water from neighboring cities in a very dry year. Growth is always anticipated to a greater or less extent in reservoir supplies, and in other water-works structures, and the chance of getting temporary supplies of inferior quality in times of great emergency. This is more practicable at the present time because of the possibility of disinfecting the water and reducing the danger of infection.

Impounding reservoirs are very stable structures, and subject to but little depreciation. If 5% is the average annual value of capital invested in them by cities, and it costs $100 per million gallons of capacity to build a reservoir larger, it will cost $5 per annum for the chance of using each million gallons of extra water when it is needed. If the storage is sufficient to maintain the service for 90% of the years, there would be a chance of selling the first additional water in one year in ten. On this basis, the costs of the water, when needed, would be $50 per million gallons, or one-half the cost per million gallons of increased reservoir capacity. In other words, if the value of the chance of using water in the driest year is worth half the cost per million gallons of building the reservoir larger,

the 90% dry year will logically be used as a basis of calculation. If the value of the chance of using the water in the dryest year is equal to the cost per million gallons of building the reservoir larger, it will pay to build for the 95% dry year, and if the chance of using the water is worth two and one-half times the cost per million gallons of building the reservoir larger it will pay to go to the 98% dry year.

There are matters which practically enter into the consideration this and are not capable of definite analysis. The beginning of the dryest year in a term, on the basis mentioned, will result in a considerable shortage of water if the whole supply is then needed, may not be distinguished in its early stages from an ordinary dry year such as recurs at frequent intervals, and does not result in a shortage of water. When the very dry year comes it may be computed that it will be possible to maintain a supply equal to 96% of the normal supply. However, if, during the first half of the period of depletion, water is drawn at the full rate, it will obviously be possible to draw water at only 90% of the full rate during the last half of that period. If the draft was continued at the full rate for three-fourths of the period, the water remaining would only suffice for 80% of the normal output for the remainder. If the extent of the dry period could be foreseen at its beginning, it would be possible to curtail the use of the water at the beginning of the dry period, so that no great hardship would result; but, if the matter of curtailing use is only taken up after most of the water in the reservoir is gone, there may be serious shortage at the end, to avoid which large expenditures would be justified.

A shortage of this kind has a far-reaching effect, and after it there will be a tendency to uneasiness in the beginning of other periods of drought, even when no shortage of water would result.

Such uneasiness may lead to the expenditure of large sums of money for temporary supplies, built in haste and not well adapted to permanent service; and such supplies may not prove to be finally necessary, for the rains will have broken the drought before they are ready for service. Such emergency developments are unfortunate, and it will be worth while to spend a certain additional sum of money to secure increased storage to. prevent the chance of needing them even though the water the water thus provided may never be required,

On the other hand it is difficult to persuade the public source of that a supply is being used to the limit. of its proper capacity while water runs over the spillway nine years out of ten, and especially when there has been no recent period of water shortage.

The matter is evidently one that must be considered broadly; and an extreme view cannot be maintained. The wisest course seems to be to take a middle basis of estimate, such as the 95% dry year, but to recognize fully its position, and to be prepared at intervals to meet a moderate shortage in supply.

Application to a Specific Case

The area of the Croton water-shed is 360.4 sq. miles, of which 19.3 are water and 341.1 are land. The estimated run-off from water area is 9.5 in. and from land area 24.0 in. For 19.3 sq. miles of water the production of water is as great as on 7.6 sq, miles of land, and the whole catchment area produces as much water as 348.7 sq. miles of land area. The total storage, including water held by flash-boards, but excluding water in the bottoms of reservoirs not available for supply, is 104 billion gallons. From this an allowance for evaporation equal to 8 in. in depth over 19.3 sq. miles must be deducted, equal to 2.7 billion gallons. The reservoirs, therefore, have a net available storage of 101 300 million gallons, equal to 290 million gallon per square mile of area. A line representing the storage is drawn across Fig. 31, and the values in Table 27 are obtained.

TABLE 27.

Year of what degree of dryness.	Maintainable yield, in thousands of gallons per square mile per day.	Total yield of catchment area, in millions of gallons per day.
80%	1 000	356
90%	970	338
95%	930	324
98%	890	312
99%	869	303

The calculation may be made in another way, using Figs.,33 and 34. The mean average run-off for the Croton River, 349 sq. miles of equivalent land area, at 1 137 000 gal. per sq. mile per day, is 397 million gallons daily, or 1,450 billion gallons per annum. The storage is 104 billion gallons. From this deduct evaporation equal to 8 in. in depth over the water area, 2.7 billions, and add for natural storage

equal to 23 days' supply at an assumed yield of 320 million gallons per day, 7.4 billions, which makes the total net storage, natural and artificial, 108.7 billion gallons, equal to 0.750 of the mean annual run-off.

Referring to Fig. 33 and interpolating between the lines for the coefficient of variation, which is 0.239 for the Croton River, it is found that in a 95% year 82.0% of the mean annual run-off should be utilized, which is equal to 326 million gallons per day. In Fig 34 it is found that in a 98% year 78.2% of the mean annual flow should be utilized, equal to 310 million gallons per day. These figures check those previously reached, and illustrate a method of computation that would be more frequently used.

It may also be seen, from the position of the point showing the quantity of flow on these diagrams, that the probable maximum time that the reservoir will not entirely fill in a 20-year period is about 5 years, being slightly greater for the larger of the two drafts and slightly less for the smaller one.

These figures mean that, assuming the exact accuracy of the data and. methods of computation, with a steady draft of 326 million gallons per day, there will probably be a shortage during some part of one year in twenty. With a like draft of 310 million gallons per day the shortage may be expected in one year in fifty, etc. The figures do not mean that there is only one chance in 50 that the supply will fall below 310 million gallons per day in any given year. They do not mean this because there are other sources of error than those thus far taken into account. Thus there are errors in the measurement of water and in the methods of calculation.

Actual Probability of Shortage in Supply

If it were required to estimate the quantity of water such that taking all these matters into account there will be only one chance in 20 or 50 that the yield in any given year will fail below it, it ought to be possible to make an approximate estimate of such amounts.

The following is suggested as the general means of procedure, and is presented as an illustration and not with a view to attach importance to the particular figures used. The foregoing figures are plotted on probability paper on Fig. 37, and the line extended crosses the 50% line at 390 million gallons per day, and this will be taken as the starting point for the calculation.

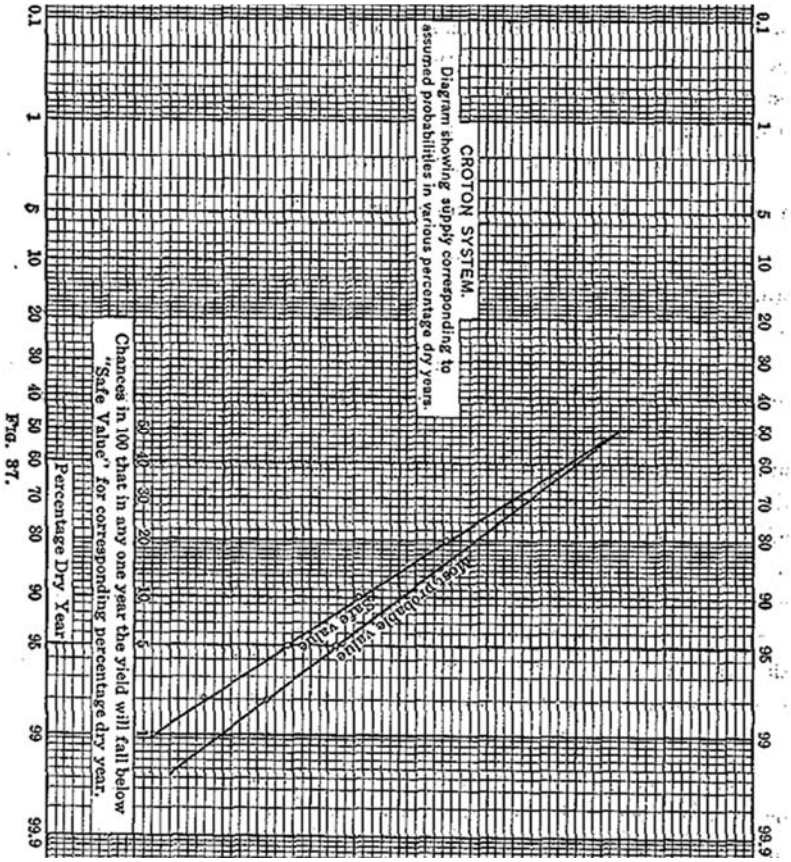


Fig. 37.

It crosses the 99% line at 299 million gallons per day. The difference between the 50% and the 99% year by the normal law of error is 3.45 times the probable error. The probable error in one term, therefore, is (390-299)/2.45 = 26.4 million gallons per day. This is equal to 6.18% of 390 millions, the starting point of the calculation. This figure may be taken as the probable error in one term of the series of annual available quantities, on the assumption that there is no error in the figure used for the mean annual flow of the stream.

The probable error in the general average for the whole series, with a standard. variation of 0.289 and a period of 45 years, is

(0.239 X 0.674)/SQRT(45) = 2.4 per cent. The probable error in the calculated mean flow, the storage being above the critical point, as a result of this error, is 0.8 of this, or 1.9 per cent.

It may be assumed that the probable error in the measurement of water is |3%, and that there are errors in calculations and methods amounting to 2 per cent. These probable errors are not cumulative, but will probably offset each other in part. The probable error of the combination will be greater than the probable error of any of the parts, but will not be equal to the sum of the probable errors of the parts. Following the basic method of ascertaining the standard variation and the probable error, it may be assumed that the probable error of the combination is equal to the square root of the sum of the squares of the separate probable errors. The probable error of the combination calculated in this way is:

$$\text{SQRT}(6.78^2+1.9^2 + 3^2 + 2^2) = 7.9 \text{ per cent}$$

For the 99% year the error will be 3.45 times the probable error, or 27.2 per cent. Deducting 27.2% from the starting point of 390 million gallons per day leaves 284 million gallons, and, on the basis taken, we should be justified in assuming that there is only one chance in 100 that the yield will fall below this quantity in any given year. Other values are given in Table 28.

Taking the 95% year as a basis, there is a 5% chance that there will be a shortage in flow in any given year with a steady use of 326 million gallons per day, this being based on the assumed accuracy of all the data and the general average of the whole term. If in addition, insurance is to be written against the errors in the general average, in measurements and calculations,

to the extent just indicated, then there would be a 6% chance that the yield would fall below 315 million gallons per day in any one year. Fig. 37 shows these two lines plotted on probability paper.

TABLE 28.

Comparative dryness of year.	Ratio of actual variation to probable error.	Total percentage variation.	Corresponding yield.
80%	1.25	9.9	351
90%	1.90	15.0	331
95%	2.44	19.3	315
98%	3.05	24.1	296
99%	3.45	27.2	284

The upper line represents the most probable conditions, and this should be used as a basis to compare different catchment areas with each other and for similar studies. With a longer record for any given area, there is certain to be some change in the position of this line, and the chances are equal that it will be moved up or down.

The lower line represents extra safe or conservative assumptions. The probabilities are that when more data are available it will be found that the maintainable yield is greater than indicated by this line, or, what is the same thing, stated in the other way, to maintain a given yield the required storage will be found to be less than shown. This line is drawn on the basis of not taking chances and resolving all doubts against the probable supply of water. It seems that this second line may be useful in the discussion of the subject. To neglect to consider it is to fail to take into account the full measure of uncertainty that there really is in the first line.

Which line should properly be used as a basis for a water supply project must be a matter of judgment, and will not be discussed at this time. It is important to point out the difference between the two bases and to urge that it shall not be overlooked; and that, whichever line is adopted, it shall be defined so clearly that there can be no misunderstanding as to what is meant.

Conclusions

The study of storage and maintainable flow of streams involves two classes of relations: First, those which are more or less definite and fixed, and can be analyzed by definite processes; second, those

which rest on conditions too complex to be understood and analyzed by ordinary means of procedure.

The variations of the second class follow, in a general way, the normal law of error, although some well-defined deviations from it have been found.

This paper presents a graphical method of reaching an approximate solution of the problem in probabilities presented by these variations, and permits definite results to be obtained from the data of stream flow and storage. These results are definite, although not exact. They may be used with confidence within limits of probable error, and these limits can be determined.

Arranging the data in this way permits results to be obtained for years of specified degrees of dryness, and thus it is possible to make studies on a more definite basis of the first above defined class of relations. It is found, also, that if the storage required to maintain a flow at all times in an average year equal to 50% of. the mean flow of the stream is determined, and if this storage is expressed in terms of days supply as the assumed rate of draft, their the increased number of days storage for higher rates of draft are approximately constant for different streams; and, on the other hand, the reduction in the number of days storage, with equal reductions in the proportionate rates of draft, are nearly the same for all streams.

In a similar manner, the increase in the days' storage, in a year so dry that only one year in twenty is dryer than it, is greater than the storage required in an average year by quantities which do not differ widely for different streams. In this way it is possible to form normal inclusive curves of storage for various rates of draft and for years of varying degrees of dryness that will apply to several streams. The actual storage required for any stream will usually be less than the normal by an amount which is approximately constant for various rates of draft and for various degrees of dryness. In this way it is possible to find how the storage required for a particular stream compares with the normal, and afterward to form a judgment of the storage required for other rates of draft and for years of varying degrees of dryness. Thus an estimate may be made which will be more accurate than could be obtained from the records of one stream, however long continued.

It has long been known that in some streams the flow is steady and in others it is variable. This fact is taken into account in this study, and a figure is used as an index of the degree of this variation. This figure, called the coefficient of variation, can be obtained from the records of flow of any stream covering a sufficient period, and, when obtained, is used as a basis for comparing the data for that stream, and estimating the probable storage and other matters in connection with relatively high rates of draft. By the use of this coefficient of variation it has been found possible to get an approximate expression for the storage required to carry the surplus water of wet years over to dry years, and to put this in such general terms that it applies almost equally well to eastern streams having the least variation in their flows, and to some western streams for which data were examined having many times greater relative variations in their flows.

The records of any one stream, even of those for which the longest records are available, are too short to establish with accuracy the probabilities of the occurrence of very dry years. The records of all the streams used in this study combined into a single series, afford a better basis for estimating the probabilities of the occurrence of very dry years than the records of any one stream. The writer believes that the basis deduced from the records of all these streams is a safer one to use than the records of any stream now having an available record, even when applied to that particular stream. This is especially true where relatively high storages and rates of draft are considered. In other words, the normal storage diagrams, as applied with suitable allowances for local conditions, are believed to afford a more reliable basis for estimating the probable yield of a stream than can be obtained by any method now available from the records of that stream alone.

The methods herein described, therefore, should afford, not only a means of interpreting more accurately the data available for streams that have been gauged for long periods, but should make it possible to make better estimates of the probable yields of streams which have not been gauged, or have been gauged only for short periods, and the performance of which must be judged, for the most part, from records of other streams similarly situated.

The precision of results reached may be disappointing to those who have made calculations of run-off and storage carried to several

decimal places such a degree of precision is not to be inferred from any run-off records at our disposal.

The use of the methods herein proposed makes it possible to estimate the probable errors in the results reached; and frank recognition of the large probable errors in many of the results cannot fail to be advantageous.

The methods of analysis herein proposed seem to be capable of application to other engineering problems, and their use wilt lead to more accurate knowledge of phenomena which contain large and unexplained elements of variation.

[The original paper is followed by 29 additional pages of discussion by 8 people.]