Optimal Hedging and Carryover Storage Value

Andrew J. Draper[1] and Jay R. Lund[2]

Abstract: Properties of optimal hedging for water supply releases from reservoirs are developed and discussed. The fundamental decision of how much water to release for beneficial use and retain for potential future use is examined analytically. Explicit correspondence is established between optimal hedging and the value of carryover storage. This more analytical view of hedging rules is useful for better understanding optimal hedging and simplifying numerical optimization of hedging operating rules. The derivations suggest the frequent optimality or near-optimality of two-point hedging policies for water supply operations.

DOI: 10.1061/(ASCE)0733-9496(2004)130:1(83)

CE Database subject headings: Water supply; Reservoir operation; Water storage.

Introduction

In reservoir operations for water supply, water can be either released for beneficial uses or retained in the reservoir for possible future use. This simple choice becomes devilishly complex in the presence of uncertain future inflows and nonlinear economic benefits for released water (Shih and Revelle 1994, 1995). The problem of how much water to withhold from immediately beneficial deliveries, retaining that water in storage, is known as ‘‘hedging’’ (Bower et al. 1962). This paper examines hedging rules analytically, deriving them from the benefits of current deliveries compared to their expected value for future uses, through reservoir carryover storage.

The literature concerning development of operating rules for water resource systems is extensive, particularly for water supplies. In general, reservoir operating rules guide release decisions. Good reservoir management therefore requires creating ‘‘a set of operation procedures, rules, schedules, or plans that best meet a set of objectives’’ (USACE 1991). Some general reviews of reservoir operating rules can be found in Lund and Guzman (1996, 1999), Loucks and Sigvaldason (1982), and Bower et al. (1962).

For water supply systems, the so-called standard operating policy (SOP) is perhaps the simplest reservoir operating rule. The SOP (Maass et al. 1962; Loucks et al. 1981) appears in Fig. 1. Reservoir release is specified as a function of the total water available currently (i.e., current storage, plus projected inflows, minus evaporation during the present period). If water supply is less than a delivery target T, all available water is released; no storage remains. Water availability exceeding the target is held in storage until at maximum capacity k the reservoir starts to spill.

In the diagram, feasible releases are constrained between two parallel lines. The upper line represents the release of all water available, with none left in storage. The lower line represents the storage of all water possible, releasing only water in excess of storage capacity. In essence, SOP places the highest priority on releasing water for immediate beneficial use, up to the level of target demand, after which remaining water available is stored until storage capacity is reached.

Hedging rules curtail deliveries over some range of water supply to retain water in storage for use in later periods (the thinner line in Fig. 1). Thus, some water is stored, rather than delivered, even when there is enough water for full target deliveries in the present period. Hedging provides insurance for higher-valued water uses where reservoirs have low refill potentials or uncertain inflows.

The intent of hedging is to reduce the risk and cost of large shortages, but at a cost of more frequent small shortages. Hashimoto et al. (1982) show that where the loss function (on releases) is linear, the SOP is the best policy. More generally, for hedging to be optimal requires a convex, nonlinear loss function (concave nonlinear benefits). Klemes (1977) found that an optimal policy converges to the SOP with increasing hydrologic or economic uncertainty. To be optimal, hedging requires not only that the loss function be convex and nonlinear but also that the hydrology have substantial probability of persistence of dry periods. Hedging would never be optimal for a hydrology that, perhaps oddly, has very severe droughts of one period followed by extremely wet conditions that always fill the reservoir. Calculation of the optimal amount of carryover storage for hedging entails an assessment and balancing of risks and costs.

A variety of hedging rules and their effects have been investigated (e.g., Klemes 1977; Stedinger 1978; Loucks et al. 1981; Hashimoto et al. 1982; Moy et al. 1986; Bayazit and Unal 1990; Shih and Revelle 1994, 1995; Srinivasan and Philipose 1996; Oliverira and Loucks 1997). The most common hedging rule forms are as follows (Lund and Guzman 1996):

- One-point hedging, where the releases begin at the origin in Fig. 1 and increase linearly (at a slope <1) until intersecting with the target level of release (Shih and ReVelle 1994),

[1]Senior Engineer, Bay Delta Office, Dept. of Water Resources, Sacramento, CA 95814. E-mail: adraper@water.ca.gov

Professor, Dept. of Civil and Environmental Engineering, Univ. of California, Davis Davis, CA 95616. E-mail: jrlund@ucdavis.edu

Note. Discussion open until June 1, 2004. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this technical note was submitted for review and possible publication on April 19, 2002; approved on January 2, 2003. This technical note is part of the ***Journal of Water Resources Planning and Management***, Vol. 130, No. 1, January 1, 2004. ©ASCE, ISSN 0733-9496/2004/1-83–87/$18.00.

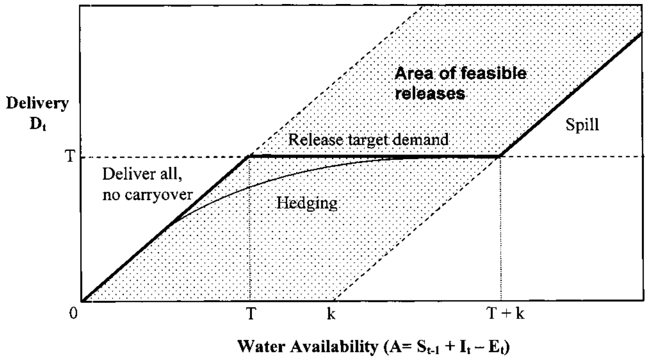

Fig. 1. Standard operating policy (thicker line)

- Two-point hedging, where a linear hedging rule begins from a first point (parameter) occurring somewhere up from the origin on the shortage portion of the SOP rule to a second point occurring where the hedging slope (<1) intersects the target release (Bayazit and Unal 1990; Srinivasan and Philipose 1996),
- Three-point hedging, where an intermediate point is specified in the above rule, introducing two linear portions to the hedging portion of the overall release rule,
- Continuous hedging, where the slope of the hedging portion of the rule can vary continuously (Hashimoto et al. 1982), and
- Zone-based hedging, where hedging quantities are discrete proportions of release targets for different zonal levels of water availability (Hirsch 1978).

Optimal Carryover Storage

The water available *A*, in the present time period, is the sum of water currently in the reservoir plus the expected value of current period inflows minus any expected reservoir evaporation or seepage losses. This water is allocated to either delivery for immediate beneficial purposes *D*, or storage in the reservoir *S*, for potential future use.

A value function typically can be specified for current water delivery benefits B(D). This value function can be economic in nature or represent some other metric of the benefits from delivering water from the reservoir for immediate use. Typically, this benefit function is concave or linear, with the marginal benefits usually decreasing with increasing use. Beyond some point d_m, there is no additional value for increasing water deliveries. Of course mathematically, deliveries are also non-negative. (Often these functions are expressed as "loss" or "penalty" functions, representing reductions in benefits from some ideal level of deliveries.) Economic benefit functions for deliveries should generally be rather smooth and convex for large water supply service areas with many consumers and heterogeneity among consumers. Water-supplying institutions and consumers usually have a large variety of water conservation and demand management options, which tend to be used in order of cost-effectiveness, leading to a generally convex economic loss function (concave benefits). Mathematically, where benefit functions are convex (losses concave), rather unusual optimal operating rules result which minimize the frequency of shortages, but when shortages are unavoidable, shortage magnitudes tend to be maximized to keep water in storage to reduce the probability of shortages in the next time-step (Hashimoto et al. 1982).

Let us also assume a value function for storing water at the end of the current decision period. This carryover storage value function $C(S)$, represents the expected value of future economic or other benefits from keeping water in the reservoir when it could otherwise have been released. The estimation of this carryover storage value function can be complex, as discussed later, but is affected by the benefit function for future use, the size of the reservoir, and the particular hydrologic patterns likely in the future. If the benefit function for use $B(D)$ is concave or linear, the economic value of storing water for the future should be concave (Gal 1979).

The economic value of carryover storage $C(S)$ is the expected value of the sum of its useful benefits discounted at rate *r* into the future [Eq. (1)], if the carryover storage *S* is partitioned into uses and losses s_t at future times *t*, such that $S=\Sigma s_t$. The value of each future use or loss of carried-over storage released in year *t* would be the marginal value of additional release at that future time $(\partial B/\partial D_t)$, multiplied by an appropriate discount factor $[\exp(-rt)]$. Losses of carried-over storage in the future from spills and evaporation create no benefit $(\partial B/\partial D_t=0)$. Future releases also may be increased by the presence of carryover storage in the reservoir from previous years. For each future time, this increased release due solely to the presence of carryover storage could be expressed as the rate of release per unit of water available times the remaining carryover storage at each future time, $(dD_t/dA_t)(S-\Sigma_{\tau=1}^{t} s_\tau)$. The estimation of $C(S)$ might not be trivial but the existence of a carryover storage value function reduces the operations problem to a deterministic equivalent form.

$$C(S)=\text{Max}\left[EV\left\{\sum_{t=1}^{\infty}\left(B_t\left[D_t+s_t+\frac{\partial D_t}{\partial A_t}\left(S-\sum_{r=1}^{t} s_t\right)\right]-B_t(D_t)\right)\exp(-rt)\right\}\right] \quad (1a)$$

or

$$C(S)=\text{Max}\left[EV\left\{\sum_{t=1}^{\infty}\left(\frac{dB_t(D_t)}{dD_t}\left[s_t+\frac{\partial D_t}{\partial A_t}\left(S-\sum_{r=1}^{t} s_t\right)\right]\times\exp(-rt)\right)\right\}\right] \quad (1b)$$

Release and carryover storage decisions should be made to maximize the sum of immediate use and carryover storage benefits. This situation can be summarized in the following simple mathematical program:

$$\text{Max}\ \ z=B(D)+C(S) \quad (2)$$

subject to

$$S+D=A \quad (3)$$

$$S\geqslant 0 \quad (4)$$

$$S\leqslant k \quad (5)$$

$$D\geqslant 0 \quad (6)$$

$$D\leqslant d_m \quad (7)$$

This formulation only applies where water available is less than maximum demand plus storage capacity $(A<d_m+k)$. If $A>d_m+k$, hedging is irrelevant since ample water exists to supply all demands, fill the reservoir, and spill, as with the SOP rule.

The Lagrangian for this problem, within the bounds of the inequality constraints where hedging is relevant, is

$$L = C(S) + B(D) + \lambda(A - S - D) \tag{8}$$

The first-order conditions for solving this problem are

$$\frac{\partial L}{\partial S} = 0 = \frac{\partial C(S)}{\partial S} - \lambda, \tag{9}$$

$$\frac{\partial L}{\partial D} = 0 = \frac{\partial B(D)}{\partial D} - \lambda, \tag{10}$$

$$\frac{\partial L}{\partial \lambda} = 0 = A - S - D \tag{11}$$

Eq. (11) gives the constraint

$$S + D = A \tag{12}$$

Eqs. (9) and (10) simplify to

$$\frac{\partial C(S)}{\partial S} = \frac{\partial B(D)}{\partial D} \tag{13}$$

In words, Eq. (13) states that at optimality the marginal benefits of storage must equal the marginal benefits of release. These conditions, Eqs. (12) and (13), can be used to derive the optimal hedging rules for a range of conditions.

Optimal Hedging Rules

Optimal hedging rules can be derived from Eqs. (12) and (13) for a variety of circumstances. If $B(D)$ is linear for $0 \leqslant D \leqslant d_m$, then hedging is not optimal under any circumstances, leaving the SOP (Fig. 1) as the optimal rule. The release rule is bound by the non-negativity of storage constraint for $A \leqslant d_m$, and then bound by $D \leqslant d_m$ (maximum usable delivery) and finally by $S \leqslant k$ (spill). Some example derivations of optimal hedging rules follow, with their implications.

Quadratic Use and Carryover Value Functions

If both $C(S)$ and $B(D)$ are quadratic functions, of the form $a_s + b_s S + c_s S^2$ and $a_d + b_d D + c_d D^2$, respectively, then combining Eqs. (12) and (13) to give optimal release D^* as a function of total water availability A gives

$$b_s + 2c_s S^* = b_d + 2c_d D^*, \text{ or } b_s + 2c_s(A - D^*) = b_d + 2c_d D^* \tag{14}$$

$$D^* = \frac{b_s - b_d + 2c_s A}{2(c_s + c_d)} \tag{15}$$

This linear form of hedging would apply in the region where inequalities (4)–(7) do not bind. This generally restricts this linear hedging rule to where

$$A > (b_s - b_d)/(2c_d) \tag{16}$$

representing where the hedging rule intersects the release of all water available boundary and

$$A \leqslant \text{Min}\left[d_m\left(1 + \frac{c_d}{c_s}\right) + \frac{b_d - b_s}{2c_s}, k\left(1 + \frac{c_s}{c_d}\right) + \frac{b_s - b_d}{2c_d}\right] \tag{17}$$

representing the hedging portion of the rule encountering the maximum useful release d_m or the release of all water remaining after filling storage capacity k constraint. Eqs. (15)–(17) result in the general form of hedging rule shown in Fig. 2, what has been called "two-point hedging."

Some interesting special cases exist. First, when $c_s = 0$, the carryover storage value function is a constant, and there exists a constant target release which may differ from that in the pure SOP rule. Second, where $b_s = b_d$ (including when $b_s = b_d = 0$), a "one-point" hedging rule results, with a constant slope from the origin.

Cubic Benefit and Carryover Value Functions

If both $C(S)$ and $B(D)$ are cubic functions, of the form $a_s + b_s S + c_s S^2 + d_s S^3$ and $a_d + b_d D + c_d D^2 + d_d D^3$, respectively, then combining Eqs. (12) and (13) to give optimal release D^* as a function of total water availability A gives

$$b_s + 2c_s S^* + 3d_s S^{*2} = b_d + 2c_d D^* + 3d_d D^{*2}, \text{ or} \tag{18}$$

$$(b_s - b_d) + 2c_s(A - D^*) + 3d_s(A - D^*)^2 = 2c_d D^* + 3d_d D^{*2} \tag{19}$$

This can be solved as a quadratic equation for D^* as

$$D^* = \frac{(c_s + c_d) + 3d_s A \pm \sqrt{(c_s + c_d)^2 + 3(d_d - d_s)(b_s - b_d) + 6(c_d d_s + c_s d_d)A + 9d_s d_d A^2}}{3(d_s - d_d)} \tag{20}$$

This results in a form of hedging that differs somewhat from the linear hedging obtained for quadratic delivery and storage value functions in Eq. (15). However, increasing the order of the value functions does not increase the order of the optimal hedging rule $D^*(A)$ proportionately. If the squared term parameters $c_s = c_d = 0$ and the linear term parameters $b_s = b_d$, then Eq. (20) becomes a purely linear hedging function of A, with only the cubed term parameters remaining. Again, the applicability of the hedging portion of the rule is restricted to the areas not bound by Eqs. (4)–(7).

Power Benefit and Carryover Value Functions

Another common form of water demand value function is the power law, where value $= qD^p$, where q and p are constants and $p < 1$ for a concave benefit function. Applying Eq. (13) with this function gives

$$q_s p_s (A - D^*)^{(p_s - 1)} = q_d p_d D^{*(p_d - 1)} \tag{21}$$

which can be solved as

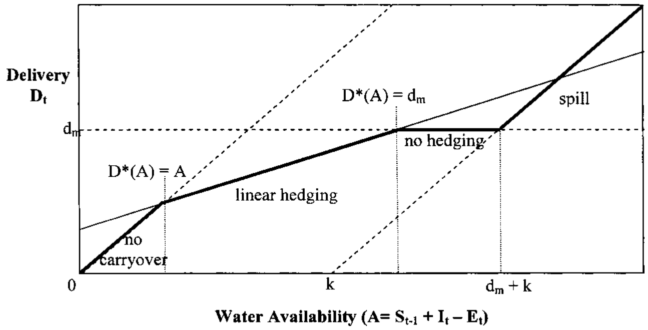

Fig. 2. Optimal hedging with quadratic use and carryover value functions

$$A=D^*+\frac{D^{*(p_d-1/p_s-1)}}{\left(\frac{q_s p_s}{q_d p_d}\right)^{(1/p_s-1)}} \tag{22}$$

For this case, the optimal hedging rule is nonlinear, but always passes through the origin.

Illustrative Example

The following examples illustrate the derivation of hedging rules from carryover storage and water demand value functions. Parameter values for quadratic, cubic, and power value functions appear in Table 1 and are plotted in Fig. 3. The constant parameters a_s and a_d are omitted since they disappear in the derivatives.

Applying Eqs. (15), (20), and (22) yield the hedging portions of the release rules appearing in Fig. 4. The applicable range of these rules is limited by where they intersect the line $D=A$, representing full release of all available water, and either the release of full demand, $D=d_m$, or full storage, $D=A-k$.

Commentary

Water resources engineers and planners are well accustomed to estimating direct benefit functions for water uses, such as $B(D)$ in this paper. Economic and noneconomic benefit functions are now commonplace in academic, theoretical, and even practical work. For water supply purposes, economic benefit functions for water deliveries are usually concave (losses convex).

Less common is the estimation of benefit functions for water storage, particularly carryover storage. Gal (1979) reasons mathematically that carryover storage value functions are monotonically increasing and concave (losses convex and monotonically decreasing). Draper (2001) recently estimated parameters for quadratic carryover storage economic value functions for several reservoir systems in California using nonlinear and grid search techniques. Carryover storage value functions were found which optimized the total economic value of system operations over a 73-year period of record. Draper found in all cases that a variety of parameter sets provided near-optimal carryover storage value functions. Under these circumstances, it seems likely that quadratic carryover storage value functions are adequate for many practical purposes. [As seen in Eq. (15), a quadratic benefit function and a linear carryover storage value function, where $c_s=0$, will lead to a SOP-type rule being optimal.] Thus, for many cases where hedging is desirable, "two-point" hedging rules appear reasonable. Draper (2001) and Howitt (unpublished) also estimate carryover storage value functions using stochastic dynamic programming (SDP) methods. SDP has some advantages in explicitly providing carryover storage value functions and including discount rates, but requires that hydrologic patterns follow one of a few probabilistic processes (e.g., Markovian), that there be good parameter estimates for these processes, and that the system can be represented with only a few reservoirs. Overall, methods for estimating optimal carryover storage value functions are not yet mature.

Where user benefit functions and optimal carryover storage value functions are available, optimal hedging rules can be derived for water supply operations. Indeed, it may be easier to

Table 1. Example Parameter Values

Value function	b, q	c, p	d	d_m	k
Quadratic					
Demand	1,000	−60	NA	8	NA
Carryover storage	800	−30	NA	NA	10
Cubic					
Demand	1,000	−35	−2	8	NA
Carryover storage	800	−30	−1	NA	10
Power					
Demand	1,000	0.7	NA	NA	NA
Carryover storage	800	0.7	NA	NA	10

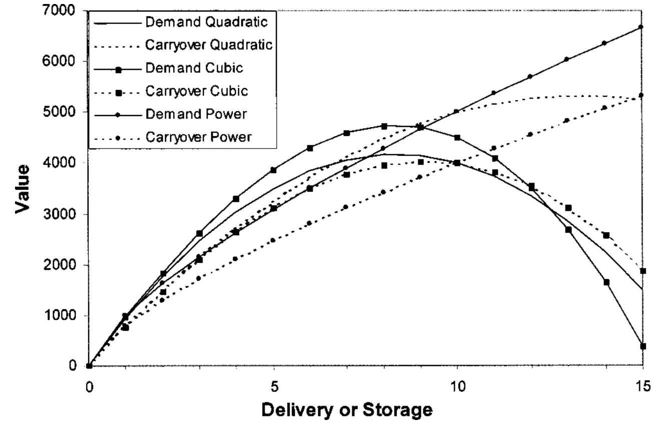

Fig. 3. Plots of example benefit functions

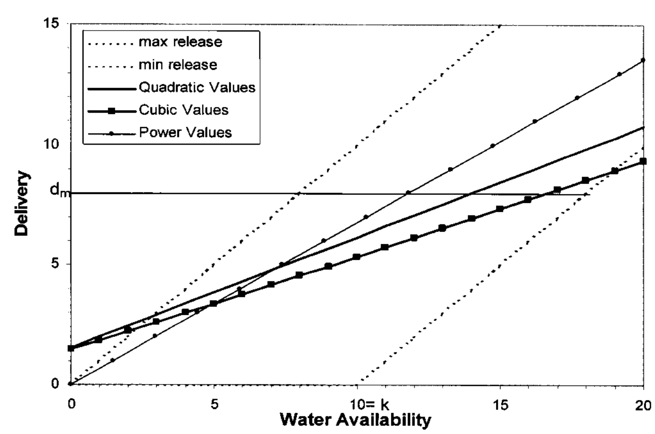

Fig. 4. Resulting hedging portions of reservoir release rules

search for the optimal carryover storage value function, and then derive the optimal hedging rule, than to search directly for the optimal hedging rule.

Where optimization is used directly to identify optimal hedging rules, often it may be adequate to examine only ''two-point'' hedging rules, which have only two parameters. This result should reduce the difficulty of direct searches for optimal hedging rules.

Of course, these theoretical results also have some limitations. Most reservoirs are not operated solely for water supply purposes. Flood control, recreation, hydropower, environmental, and other uses further complicate real operating rule studies. Even where these benefit function complications are surmountable, estimation of optimal carryover storage value functions can remain challenging, particularly for more complex multi-reservoir systems (Draper 2001).

Conclusions

This paper demonstrates that the optimal hedging policy for water supply reservoir operations depends on a balance between beneficial release and carryover storage values. Optimal hedging policies can be derived for a given pair of beneficial delivery and carryover storage value functions. This provides an analytical view of hedging rules and operations.

Given that quadratic carryover storage value functions may fit a range of reservoir operations settings well (Draper 2001), it seems likely that where hedging is desirable, a linear ''two-point'' hedging policy may be near optimal for a wide range of circumstances. Even where a third-order carryover storage function is optimal, the optimal hedging policy might not deviate greatly from a ''two-point'' linear policy.

Acknowledgments

Thanks to Mimi Jenkins for her comments on an earlier version of this paper and two anonymous reviewers for their suggestions and corrections.

References

Bayazit, M., and Unal, N. E. (1990). ''Effects of hedging on reservoir performance.'' *Water Resour. Res.,* 26(4), 713–719.

Bower, B. T., Hufschmidt, M. M., and Reedy, W. W. (1962). ''Operating procedures: Their role in the design of water-resource systems by simulation analyses.'' *Design of water resources systems*, A. Maass, M. M. Hufschmidt, R. Dorfman, H. A. Thomas Jr., S. A. Marglin, and G. M. Fair, eds., Harvard University Press, Cambridge, Mass.

Draper, A. J. (2001). ''Implicit stochastic optimization with limited foresight for reservoir systems.'' PhD dissertation, Dept. of Civil and Environmental Engineering, Univ. of California, Davis, Calif.

Gal, S. (1979). ''Optimal management of a multireservoir water supply system.'' *Water Resour. Res.,* 15(4), 737–748.

Hashimoto, T., Stedinger, J. R., and Loucks, D. P. (1982). ''Reliability, resilience, and vulnerability criteria for water resource system performance evaluation.'' *Water Resour. Res.,* 18(1), 14–20.

Hirsch, R. M. (1978). ''Risk analyses for a water-supply system—Occoquan Reservoir, Fairfax and Prince William Counties, Virginia.'' *Open-File Rep. 78-452*, U.S. Geological Survey, Reston, Va.

Klemes, V. (1977). ''Value of information in reservoir optimization.'' *Water Resour. Res.,* 13(5), 857–850.

Loucks, D. P., and Sigvaldason, O. T. (1982). ''Multiple-reservoir operation in North America.'' *The operation of multiple reservoir systems*, Z. Kaczmarck and J. Kindler, eds., IIASA Collab. Proc. Ser., CP-82-53, 1–103.

Loucks, D. P., Stedinger, J. R., and Haith, D. A. (1981). *Water resources systems planning and analysis*, Prentice-Hall, Englewood Cliffs, N.J.

Lund, J. R., and Guzman, J. (1996). *Developing seasonal and long-term reservoir system operation plans using HEC-PRM*, Technical Report RD-40, Hydrologic Engineering Center, U.S. Army Corps of Engineers, Davis, Calif.

Lund, J. R., and Guzman, J. (1999). ''Derived operating rules for reservoirs in series or in parallel.'' *J. Water Resour. Plan. Manage.,* 125(3), 143–153.

Maass, A., Hufschmidt, M. M., Dorfman, R., Thomas, H. A., Jr., Marglin, S. A., and Fair, G. M. (1962). *Design of water resources systems*, Harvard University Press, Cambridge, Mass.

Moy, W.-S., Cohon, J. L., and ReVelle, C. S. (1986). ''A programming model for analysis of the reliability, resilience, and vulnerability of a water supply reservoir.'' *Water Resour. Res.,* 22(4), 489–498.

Oliverira, R., and Loucks, D. P. (1997). ''Operating rules for multireservoir systems.'' *Water Resour. Res.,* 33(4), 839–852.

Shih, J.-S., and ReVelle, C. (1994). ''Water supply operations during drought: Continuous hedging rule.'' *J. Water Resour. Plan. Manage.,* 120(5), 613–629.

Shih, J.-S., and ReVelle, C. (1995). ''Water supply operations during drought: A discrete hedging rule.'' *Eur. J. Oper. Res.,* 82, 163–175.

Srinivasan, K., and Philipose, M. C. (1996). ''Evaluation and selection of hedging policies using stochastic reservoir simulation.'' *Water Resour. Manage.,* 10, 163–188.

Stedinger, J. R. (1978). ''Comment on 'Value of information in reservoir optimization' by V. Klemes.'' *Water Resour. Res.,* 14(5), 984–986.

United States Army Corps of Engineers (USACE). (1991). ''Optimization of multiple-purpose reservoir system operations: A review of modeling and analysis approaches.'' *Rep. RD-34*, Hydrologic Engineering Center, Davis, Calif.